

ThinQ
think • inquire • question

A THEORY OF MOTION*

K P Mohanan and Tara Mohanan

18 March 2024



* This work is part of

THE FIG TREE PROJECT

funded by

the late Dr Sunita Anne Abraham

PhD, University of Birmingham, UK 1993



CREATIVE COMMONS: ATTRIBUTION - NON COMMERCIAL - SHARE ALIKE

[This license allows others to remix, adapt, and build upon this work, as long as they **credit the authors**, use it **non-commercially**, and **license their new creations under the identical terms.**]

NOTE: If you find any errors, please alert us at:
tara.mohanan@gmail.com and mohanan.kp@gmail.com

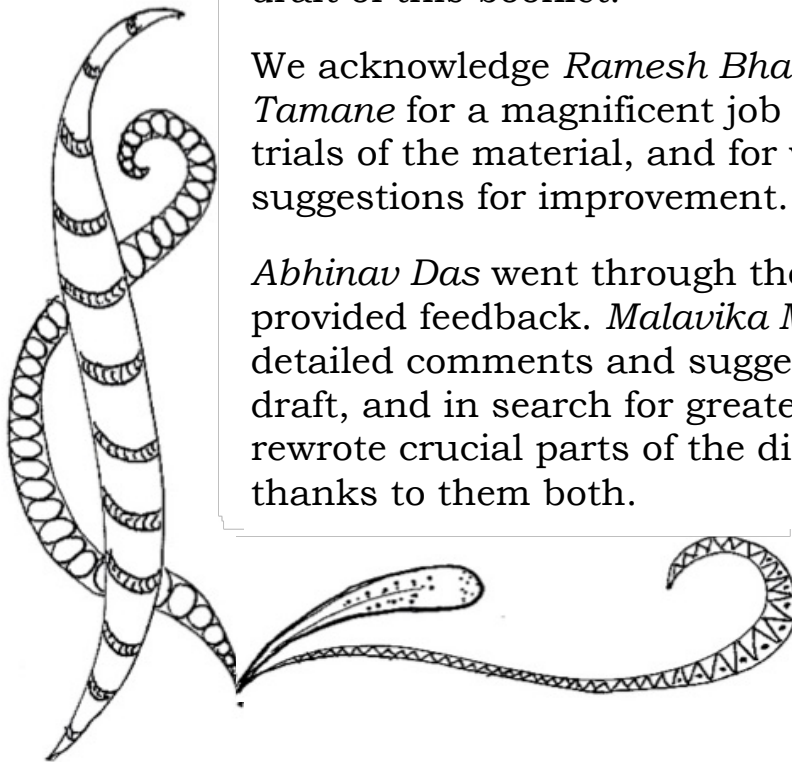
ACKNOWLEDGEMENTS

This booklet owes its existence to the generous gift to ThinQ from our dear friend, Sunita Abraham. While Sunita is no longer with us, and we have no words to express our deep gratitude to her, she will live on in the project that was born out of her gift — the project of creating learning materials for children — the Fig Tree Project.

We are grateful to *Sriram Naganathan*, *Jayasree Subramanian*, and *Rashmi Jejurikar*, for extensive feedback on an earlier draft of this booklet.

We acknowledge *Ramesh Bhaskar* and *Sayali Tamane* for a magnificent job of doing field trials of the material, and for valuable suggestions for improvement.

Abhinav Das went through the draft and provided feedback. *Malavika Mohanan* gave detailed comments and suggestions on a draft, and in search for greater coherence, rewrote crucial parts of the dialogues. Our thanks to them both.



TO THE READER

We hope that this book will help you to appreciate what science is; give you a glimpse into the way scientists think and the kinds of questions they ask; and help you to develop the capacity to think like a scientist.

Why is this important? Well, the abilities you develop in the process will be useful in other domains of learning, and also in life beyond formal education.

Let us be a bit more specific. This is an introduction to how we construct theories in science, and how we evaluate those theories. To construct a theory, we need some specific subject matter. Here, the subject matter is motion, studied in physics. But the essence of what we explore here through the study of motion would form the foundations for constructing theories in all domains of academic knowledge. And on the side, you might also gain a better understanding of some of the foundational concepts in physics.

The content knowledge of science and mathematics needed for making the most of this book is what you have probably learnt in school, perhaps in Class 8. We believe that the book would be of some value to even undergraduate and graduate students, and to PhDs in physics or, for that matter, in any other subject.

This is not a typical textbook. Its aim is not to ‘communicate’ the latest scientific knowledge to you, to make science popular, or to convey ideas in science in a journalistic way. To get the most out of the book, you need to work through the learning tasks in each section, the way you would do in a conventional physics textbook. There is a difference though. In this book, you wouldn’t be calculating, solving numerical problems, or deriving results from equations and formulae. Instead, you would wrestle with understanding the concepts, the ideas, and the reasoning that you come across, to get your mental muscles strong and agile enough to be able to play with even more new concepts and ideas.

If you find that path inspiring, we welcome you to join us on this exciting intellectual adventure!

A NOTE ABOUT THE STRUCTURE OF THIS BOOK

This book takes the form of dialogues in various situations among three 14-year old students, twins Anju and Zak, and their friend, Ebin. They are children with properties of the mind that are characteristic of a scientist — imagination, insight, intuition, reasoning, rigour, clarity of thought and expression, and above all, intellectual curiosity.

As you read, imagine that you are one of the characters, try to think like them, and join them in imaginary conversations.

In the book, you will find activities that require you to explain certain puzzling phenomena, and to design and implement experiments to test your explanations. Try to engage in these activities, on your own as well as in small groups. You might discover that you learn as much from your group members as you do from listening to lectures or reading textbooks.

The units in this book is accompanied by a section for the advanced reader, which appear at the end of the book under the title, ‘Appendices for Advanced Readers’.

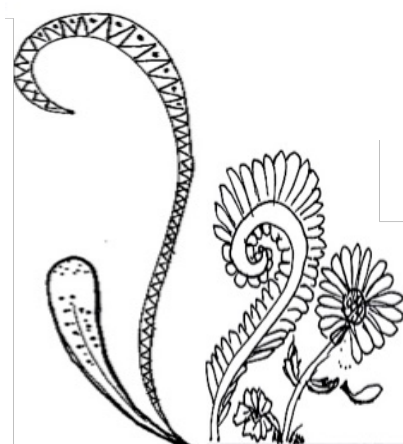
By ‘advanced readers’, we mean anyone — including teachers — who has background in physics beyond what is learnt in high school.

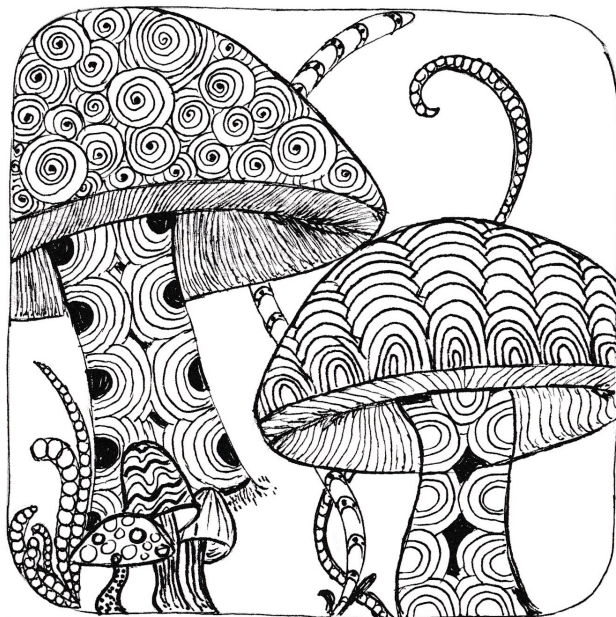
A THEORY OF MOTION

CONTENTS

Acknowledgements	3
To the Reader.....	4
A Note about the Structure of the Book	5
CHAPTER 1: MOVEMENT AND FORCE	9
1.1 Motion: A Regularity	
1.2 More Regularities: When Objects Collide	
1.3 Force as a Cause of Motion	
1.4 The Direction of Force and of Movement	
CHAPTER 2: MOVEMENT AND TRANSFORMATIONS	21
2.1 Angle of Incidence	
2.2 Displacement, Rotation, and Revolution	
2.3 Identifying the Essentials	
2.4 Theory as a Configuration of Laws	
CHAPTER 3: SEESAWS AND LEVERS	33
3.1 The Behaviour of Seesaws	
3.2 The Behaviour of Levers	
CHAPTER 4: SLIDES AND INCLINED PLANES	41
4.1 Slides and Swings	
4.2 A Dropped Stone Moves Downwards. Why?	
4.3 Action at a Distance	
4.4 Two Competing Theories of Motion	
4.5 Force Causing Change of Speed	

CHAPTER 5: SWINGS AND PENDULUMS	49
5.1 The Behaviour of Swings	
5.2 A Pendulum's Amplitude and Speed	
5.3 Designing the Experiment	
5.4 Predicting the Period of a Pendulum	
5.5 Collision and Gravity	
CHAPTER 6: HOW SCIENCE PROGRESSES	61
CONCLUDING REMARKS	67
APPENDICES: FOR THE ADVANCED READER	69
1 Doubting and Questioning	70
2 Physics of Motion before Galileo	72
3 Archimedes' Levers: Math or Science?	72
4 Galileo and the Pendulum	73
5 From Verbally Stated Laws to Equations	75
6 Is Friction a Force?	77





CHAPTER 1

MOVEMENT AND FORCE

1.1 Motion: A Regularity

Anju was at the dining table, reading a book. When she looked up, she thought she saw a lump of something at the far corner of the table. It looked like a ball of dirt. She didn't pay further attention, and went back to reading. Soon, while turning a page, she happened to look up again. The lump was now in a different place. She glanced at the spot again, decided that her memory was playing tricks, and went back to reading. When she looked up again a few seconds later, the lump was in yet another spot on the table.

Curious, Anju started looking attentively at the lump. She found that it was moving, very very slowly. Amazing! How can that be? She called out to her brother Zak before realizing that he had gone out for basketball practice. Just then, the phone rang. It was their friend Ebin.

Anju: Ebin, there is something on my table that looks like a dirtball, but it's moving! VERY slowly!

Ebin: Are you sure it is a dirtball? It could be some kind of creature.

Anju: No, I am sure it is not a living thing.

Ebin: Take a careful look, Anju. Do you have a magnifying glass?

Anju: Yes, I do. Wait a minute.

Anju got her magnifying glass from her desk drawer, turned on more lights, and looked at the lump. It had what looked like legs!

Anju: You were right, Ebin! Looks like it is indeed a creature.

Ebin: How do you know that?

Anju: It has legs! And it's walking on them. You know what? I was surprised earlier when I saw it move because I assumed it was not a living thing.

Ebin: You thought it was inanimate, not a living creature.

Anju: Mm-hm. And inanimate things don't move on their own, right? That's why I was puzzled.

Ebin: Is that ALWAYS so, what you just said about inanimate entities?

Anju: Come to think of it, it's always so. We can say this for sure:

“Inanimate entities do not move on their own.”

Ebin: Move on their own? What do you mean by that?

Anju: Take a stone. It moves only when something makes it move. But an ant or a bird can move even when there is nothing from outside making it move.

Ebin: Okay, *inanimate entities do not move on their own*. That sounds like a **principle**.

Anju: What do you mean, principle?

Ebin: Well, you noticed a pattern; that certain things follow certain rules, **all** the time, with regularity. In this case, the regularity you noticed is that inanimate things don't move on their own. And that is what you stated. I think a principle, or a law of nature, is just the statement of a regularity.

Anju: Oh, okay. So, I noticed a pattern and stated it as a principle. But then I noticed something that didn't fit the pattern. That's why I was puzzled.

Ebin: So it goes against your principle, right? Then it is a **counterexample** to your principle.

Anju: I guess... I thought that the lump of dirt on the table was inanimate. There was no external push or pull acting on it, and yet it was moving. That puzzled me. But once I saw that it was a living thing, it was not a counterexample any more, so it wasn't puzzling.

Ebin: Great! Do you realize you're a budding scientist, Anju?

Anju was pleased at Ebin's complement.

Ebin: Listen, how about if we state the principle a bit differently?

*“Inanimate entities move only when
an external push or pull makes them move.”*

STOP AND THINK 1.1

Think about what you have read so far. All clear? If you have no questions, attempt the task below, before you read any further.

Anju stated a **principle**, and Ebin restated it a bit differently.

Do you think the principle is true?

Can you think of any **counterexamples**?

When Zak came home from basketball practice, Anju told him about her conversation with Ebin and the principle of motion they had formulated.

Zak: But Anju, there are so many counterexamples to that principle.

Anju: Like what?

Zak: A wristwatch is inanimate, right? But its hands move. The same with a toy car; if you wind it up and place it on the floor, it moves. What about rockets and robots? They are inanimate too, but they move without any external push or pull acting on them.

Anju: Agreed. But all your examples are of machines. They are manmade, not natural. The principle states a pattern we find in nature.

Zak: Then the principle has to say so. The way you guys have stated it, does it say that it applies only to natural objects?

Anju: (after a moment's thought) You're right. What if I revise it like this?

In her notebook, Anju wrote a version of the principle that was further revised:

“Inanimate entities that are not manmade move only when an external push or pull makes them move.”

Zak: That might work.

Anju: So you accept that the statement is true?

Zak: No, no, no, I didn't say that. I just mean that for now, TILL we bump into counterexamples, let's take the principle to be true.

1.2 More Regularities: When Objects Collide

The next day at school, Anju and Zak told Ebin briefly about their revision of the principle of motion that Ebin and Anju had stated earlier. Ebin, as usual, listened carefully. He was silent for a while, and then got up.

Ebin: Let's go back to the classroom.

Zak: But why? There's a lot of time before the bell.

Ebin: We might need the whiteboard.

Zak: Oh, okay.

Ebin: What makes inanimate objects move? If I swing my leg, and my foot hits a ball, the ball would roll away, right?

Zak: That's right. So your foot hitting the ball is the external push that makes the ball roll away.

Ebin: And if that ball hits another ball as it rolls, the second one would also start rolling.

Zak: So the moving ball hitting the stationary ball is another external push.

Anju: This means we need to state another principle.

Ebin: How about this?

“When a moving object collides with a stationary object, the moving object serves as the external push to make the stationary object move.”

Anju: Cool!

Ebin: Wait! We’re missing something. The faster I swing my leg to hit the ball, the faster the ball would roll. Right?

Zak: Hmm! You’re right. I hadn’t thought of that. This means there must be other regularities in the way motion happens.

Anju: So we’ll need to state more principles of motion.

They were in the classroom now. Anju picked up a whiteboard marker.

Anju: Let me write down the principles we already have.

She wrote:

PRINCIPLES OF MOTION (PMs)

PM-A: Inanimate entities that are not manmade move only when an external push or pull makes them move.

PM-B: When a moving object M collides with a stationary object N, M serves as the external push to make N move.

Zak: Looks like a good start.

Ebin: Right! My mom was saying that as we explore, we’ll find more patterns that will need to be stated as principles. And when we have a bunch of related principles like that, she said, we might end up with a *THEORY* of motion!

Anju: Ebin, is your mom a teacher? She talks like one. Did you understand what she said to you?

Ebin: Of course I did. Oh, well, at least I think I understood it.

Anju: Can you tell me what you think you understood, so that I can say I think I understand?

Ebin: Let me try. She was saying that a scientific theory is an explanation for something that is puzzling. You were puzzled when you saw that thing move, right, when you thought it was a dirtball?

Anju: M-hmmm.

Ebin: What puzzled you when you thought it was a dirtball? And how come your puzzlement evaporated when you realized it was a creature? If you answer those two question, you get a scientific theory.

Anju: Ah, I see.

Ebin: My mom was saying that a scientific theory is a bunch of principles that give you explanations for what is puzzling in what you see.

Anju: WoW! Your mom's great! So that's what a theory is?

Ebin: I suppose so.

Zak: Sounds exciting.

STOP AND THINK 1.2

Can you think of counterexamples to PM-B above?

A *COUNTEREXAMPLE* here would be an example where a moving object collides with a stationary object, but that object still does not move. If there is indeed such a case, you need to revise the statement, or add another statement, so that there are no counterexamples.

1.3 Force as a Cause of Motion

As Ebin rode his bicycle to school the next day, he was preoccupied. Something was bothering him. He was glad to see that Anju and Zak were already at the school when he arrived.

Ebin: Remember the principles of motion from yesterday?

Zak: Of course.

Ebin: I am concerned about the second principle. It may not work.

Zak: How so?

Ebin: If the moving object is light, and the stationary object is really heavy, the stationary object won't move on collision. Suppose a moving table tennis ball hits a big iron ball that is stationary.

Anju: Table tennis? Tennis on a table?

Zak: No, silly. Only the ball is on the table, not the players. You know ping pong, right? That's just the popular name for table tennis.

Ebin: Okay, to go back, the iron ball is not going to move when a table tennis ball hits it. If a moving bicycle collides with a stationary truck, the truck is not going to move.

Anju: Oh, right! So the principle has to say when the stationary object would move and when it wouldn't move.

Ebin: Exactly.

The bell rang just then, and they had to go to their class. All morning, Anju's mind was on the principles of motion. This kept her from paying attention to what was happening in class. As soon as the bell rang for the break, she ran out to talk to Zak and Ebin.

Anju: Listen, when I want to open a bottle, if the lid is closed tight, and I try to turn it, it doesn't move. But Zak can open it. He is stronger than me, so he can twist the lid off easily.

Zak: So? What's your point? Why are you saying this?

Ebin: I think I see where she is going with this. She is saying, the tight lid resists change. I wonder if that is what my mom calls inertia.

Anju: What does it mean to say, the lid resists change?

Ebin: You are rotating the lid when you turn it, right?

Anju: Rotating?

Ebin: Yep. Suppose you draw a line on the lid from its center to its edge. Now turn the lid. You can see the line changing its direction, right? That is change of orientation. And rotation is change of orientation.

Anju: Oh, okay, I got that one. You said something about inertia. What is that?

Ebin: When you hit a ball, it moves. It is not in the same location any more, Like your moving dirt-ball that changed its location. So you can say that motion is *change of location*, just like rotation is *change of orientation*. Now, when you use force to change something, in some cases the change doesn't happen, like what happens when you push a truck to move it, or try to open a rusty lid. That effect of not moving? That resistant to change? I believe that is what Mom calls inertia.

Anju: I suppose we can say that the tight lid resists change.

Zak: You're right, Ebin. We can think of inertia as resistance to change. It's what keeps me from getting out of bed early in the morning, unless there is the pull of a basketball game.

Anju: Aha! So force is what causes change. When we pull, push, twist, or lift something, we are using force. So pushing, pulling, and twisting are examples that involve force.

Ebin: Right. And inertia resists change, a kind of opposing force.

Anju: Good. I'm going to add two more principles to the ones we have.

Zak: Let's hear them first.

Anju: One is what we just said. Force causes change.

Zak: And the second one?

Anju: Inertia is resistance to change.

Ebin: Any more?

Anju: As I said, Zak can open a jar that I can't. He is stronger so he can use more force. So maybe for the lid to move, the force that causes the movement has to be more than the lid's resistance to change. So here is the third one: the greater the inertia of an object, the greater the force needed to cause a change.

Ebin: Got it. Nice!

Anju wrote on the whiteboard, replacing the earlier PM-A with a specification of the scope of the theory (that it applies only to inanimate objects), and PM-B with a new set:

PRINCIPLES OF MOTION OF INANIMATE OBJECTS (PMs)

PM-1: *Force causes change: of location, or of orientation.*

PM-2: *Moving objects have force. When a moving object collides with a stationary object, the force of the moving object causes the stationary object to move.*

PM-3: *Inertia is the resistance to change. All objects have inertia.*

PM-4: *The greater the weight of an object, the greater is inertia.*

PM-5: *The greater the inertia of an object, the greater the force needed to make it change.*

Ebin: That looks good, Anju! But it is not just a matter of heavy and light. It also depends on the *kind* of movement. So take a really heavy suitcase. It is easier to push it horizontally than to lift it. So the suitcase has greater inertia against upward motion than against horizontal motion.

Anju: Yes. Another example is the lid on a bottle. A lid that is tightly screwed on has more inertia than the same lid loosely screwed on.

And it is more difficult to twist off. So, the tighter the fit, the greater the inertia.

Ebin: Sounds reasonable.

Anju: I just thought of something. Principle 5 says that the greater the inertia of something, the greater the force needed to change its location or orientation. So pushing the heavy suitcase across a smooth floor takes less force than pushing it across a rough floor. And if the suitcase has wheels, it takes even less force to move it.

Zak: Ebin, you had an example earlier of a moving table tennis ball colliding with a big iron ball that is stationary.

Anju: Well, the iron ball doesn't move because its inertia is much greater than the force of the moving table tennis ball.

Zak: But how do you find out what the inertia of a stationary object is, and what the force of a moving object is? If we don't have a way to find that out, we can't tell if the principle holds.

Anju: Let's see. I think there are three parts to this. The first has to do with weight. The heavier an object, the greater its inertia. The second has to do with the force of a moving body causing change in some other body. The greater the inertia of a body, the greater the force needed to change its location or orientation. And the third point is, the faster the motion of a body, the greater its force.

Zak: Our principles already include all three ideas.

Ebin: But how do we find out how much inertia a body has?

Zak: I have an idea. Imagine three balls of the same size. One is made of lead, another of wood, and the third of stone. Suppose we roll the lead ball, and make it hit the wooden ball. And then we make the stone ball, moving at the same speed, hit the wooden ball. According to point 2, since the lead ball is heavier than the stone ball, it would hit the wooden ball with greater force.

Ebin: That sounds reasonable. But how do we know if the lead ball has hit with greater force? We can't perceive force. Does hitting with greater force result in something that we *can* perceive? What would the consequence of 'greater force' be?

Anju: One consequence of greater force would be greater speed. When you watch a game of golf, you can see that the greater the force with which you hit the golf ball, the faster it goes.

Zak: Ah, so we need to add yet another principle:

PM-6: The greater the force acting on a body, the faster it changes its location or orientation.

Anju: So one consequence of greater force is greater speed. We can perceive it, and also measure it. When the lead ball and the stone ball hit the wooden ball, since the lead ball is heavier, it should result in the wooden ball moving faster. IF that is what happens, we must conclude that the lead ball hits the wooden ball with greater force than the stone one, right?

Ebin: Great!

Zak: Suppose we hit the wooden ball with the lead ball several times, changing the speed of the lead ball. By point 3, the greater the speed with which the lead ball hits the wooden one, the faster the wooden ball should move. If this turns out to be true, then we know that point 3 also is on the right track.

Anju: This must be why when a moving vehicle hits another vehicle, the greater the speed of the moving one, the greater the damage.

Zak: I want to go back to the three balls. It follows from the theory that the wooden ball should move faster when it is hit by the lead ball than when it is hit by the stone ball. If it happens the other way round, our statements are false.

Ebin: Wow! That is like a mathematical proof. Cool! But I need to go through this by myself to make sure that the prediction does follow.

Zak: I think we have enough principles to call it the beginning of a theory of motion.

Ebin: That's right! Our principles are connected, and when we put them together, what they predict matches what we observe. They explain what we observe. So yes, a theory is taking shape.

1.4 The Direction of Force and of Movement

On his way home, Ebin was still thinking about the theory of motion that was emerging through their discussion. The first two principles said:

PM-1 Force causes change: of location, or of orientation.

PM-2 Moving objects have force. When a moving object collides with a stationary object, the force of the moving object causes the stationary object to move.

But was that precise enough? he asked himself. What about the direction of motion? Suppose a moving ball X collides with a stationary ball Y. Would Y move perpendicular to the direction in which X was moving? No! Would it move in the opposite direction? No! So what was missing in their statement? We need yet another principle! he thought.

Just as he got home, a statement popped up in his mind. He pulled out his notebook and wrote:

PM-7: When a moving object causes another object to move, the motion of the second object is in the same direction as the motion of the first object.

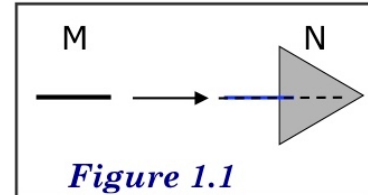
He called Anju and Zak to tell them about the new principle. The three of them spent a long time looking for counterexamples, but couldn't find any. So they decided to accept the principle as true till they found evidence to show that it was false.

But the next day, Zak pointed out a problem with PM-7.

Zak: Look, Ebin, given our principle 7 it follows that when X hits Y, Y does not move either perpendicular to X's motion or in the opposite direction. I agree, that's a good generalization. But the principle says that the two move in the same direction. Isn't it possible that the second object moves at an angle to the direction of the motion of the first one?

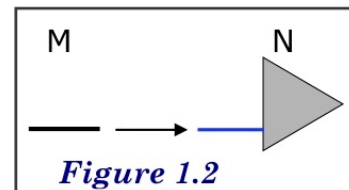
Ebin: Is that possible?

Anju: I think so. Imagine a wooden equilateral triangle. Suppose we place it flat on a table, and hit it at the center of one of its sides with a thin straight stick such that the line through the stick, when extended, passes through the centre of the triangle (Fig. 1.1). The stick would be perpendicular to the side of the triangle. The triangle will move in the same direction as the stick, right?



Ebin: Yes.

Anju: But if the stick hits the triangle off center (Fig. 1.2), still perpendicular to the side, would the triangle move in the same direction as the stick?



Ebin: I think I see what you mean. It would either move off at an angle, or it might rotate a bit.

Anju: Shouldn't we revise the principle like this?

PM-7: When a moving object causes another object to move, the second one moves in the same direction as the movement of the first, or at an angle to that direction.

Zak: I think that would work.

Here is the theory of motion that Anju, Zak and Ebin have come up with so far, either as points in their discussions, or as explicitly stated principles:

A THEORY OF MOTION OF INANIMATE OBJECTS

- PM-1: *Force causes change: of location, or of orientation.*
- PM-2: *Inertia is the resistance to change. All objects have inertia.*
- PM-3: *The greater the weight of an object, the greater its inertia.*
- PM-4: *Moving objects have force.*
- PM-5: *When a moving object X collides with a stationary object Y, the force of X causes Y to move.*
- PM-6: *The greater the inertia of an object, the greater the force needed to make it change.*
- PM-7: *The greater the weight of a moving object, the greater the force with which it acts on a stationary object.*
- PM-8: *The greater the speed of a moving object, the greater the force with which it acts on a stationary object.*
- PM-9: *The greater the force acting on a body, the faster it changes its location or orientation.*
- PM-10: *When a moving object X causes a stationary object Y to move, Y moves either in the same direction as the movement of X, or at an angle to that direction.*

STOP AND THINK 1.3

Can you find counterexamples to the set of Principles of Motion 1-10?

CHAPTER 2

MOVEMENT AND TRANSFORMATIONS

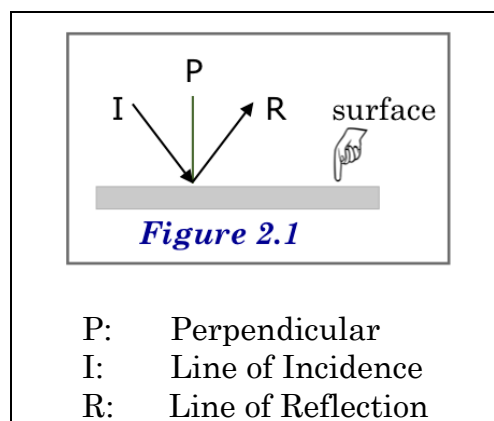
2.1 Angle of Incidence

Zak was lying awake in bed that night, his mind on the PMs (principles of motion) that he, Anju, and Ebin had come up with, when something struck him. *If we drop a rubber ball from a height, it falls perpendicular to the floor, hits the surface of the floor, and bounces up, perpendicular to the surface. But if we throw the ball at an angle, it bounces up at an angle, not perpendicular to the surface.*

This reminded him of the term *angle of incidence*. He had learnt it as part of the lesson on the behaviour of light. When a particle of light falls on a mirror, it bounces off in such a way that the angle of incidence is equal to the *angle of reflection*. He tried to recall what he had learnt.

The ***angle of incidence*** is the angle between the perpendicular to the surface of the mirror, and the direction in which the particle hits the mirror.

And the ***angle of reflection*** is the angle between the perpendicular and the direction in which the particle bounces off the mirror.



Zak was excited. Perhaps this idea of bouncing of balls could be extended from mirrors to other surfaces, and from rays to solid objects. It might help him and Anju, and Ebin too, to better understand the motion of colliding objects. For now, he took an extended version of what he had learnt about the angle of incidence and angle of reflection as definitions of those concepts:

Angle of Incidence (DEF):

The angle of incidence of X is the angle between the direction in which X hits a surface and the perpendicular to that surface.

Angle of Reflection (DEF):

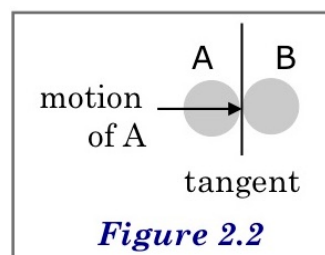
The angle of reflection of X is the angle between the perpendicular to a surface and the direction in which X bounces off that surface.

Using these definitions, he added a second part to PM-10, as PM-10b:

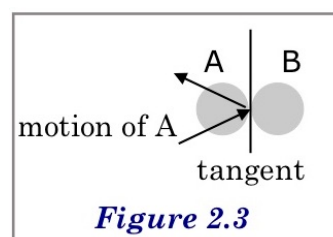
PM-10a *When a moving object X causes a stationary object Y to move, Y moves either in the same direction as the movement of X, or at an angle to that direction.*

PM-10b *When a moving object X collides with the flat surface of a body T such that X bounces off T, the angle of incidence is equal to the angle of reflection.*

What would happen if both A and B were spheres, he wondered. If the line of motion of A passes through the center of B, then the line of motion is perpendicular to the tangent of B. In that case, B would begin to move in the same direction, and A would simply bounce off in the opposite direction.



But what if A's motion is not perpendicular to the tangent? In that case, when A bounces off, the angle of reflection would be equal to the angle of incidence.



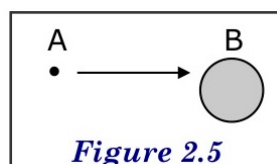
Zak was thrilled by what he had figured out.

2.2 Displacement, Rotation, and Revolution

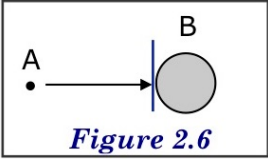
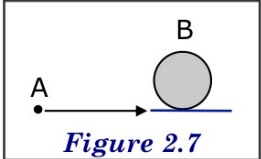
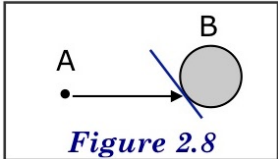
As Zak got out of bed the next morning, his thoughts again went back to the PMs. It occurred to him that perhaps the nature of motion depended on the size of the objects. He took out his note-book and started drawing pictures. Suppose A and B are both small beads (Fig. 2.4). If A moves towards B and collides with it, the collision would cause B to move. B would move in the same direction as A, not in any other direction.



But what if A is a small bead, and B is a big ball (Fig. 2.5)? We have three possibilities:



The path of A:

<p>I. when extended, passes through the center of B.</p>  <p><i>Figure 2.6</i></p> <p>B moves in the same direction as A.</p>	<p>II. is along the edge of B and forms a tangent to it.</p>  <p><i>Figure 2.7</i></p> <p>B rotates in the same location; it does not change its location.</p>	<p>III. lies between the center of B and its edge.</p>  <p><i>Figure 2.8</i></p> <p>B rotates, and moves at an angle to the path of A.</p>
--	---	---

These figures set Zak wondering further. What if the scenario is reversed — A is a big ball, and B is a small bead. What would happen if A moved towards B and collided with it?

STOP AND THINK 2.1

Can you help Zak figure out the answer to his question?

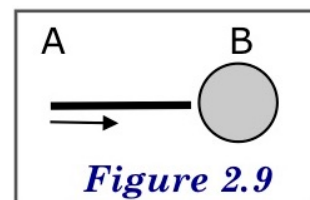
STOP AND THINK 2.2

Now consider this scenario. What would be the result if both A and B are big balls, and A collided with B?

Zak felt energised. New questions were emerging. He called out to Anju and told her about them.

Anju decided to test Zak’s ideas and check the results. But she couldn’t find any big balls that she could use for the experiment. “How about if we use disks instead of balls? In any case, measuring the angles is easier if A and B are disks, not spheres,” she said. She took out their carrom board, the striker, and a carrom piece, which was smaller than the striker. She placed the striker at the center of the board. She then placed the carrom piece at the edge of the board, and struck the striker with it. The carrom piece moved and collided with the striker. But it was too fast for them to see if the situation was as in scenario I, scenario II, or scenario III.

Zak said: “What if instead of a carrom piece, we use a straight thin rod to hit the striker, so that we can control the angle at which the rod (A) hits the striker (B)? (Fig. 2.9) Then we can also control, with greater precision, the direction of the movement of the rod, and where it hits the disk.



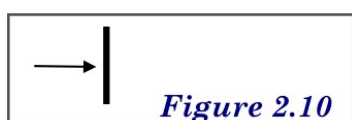
Anju used a metal rod to hit the striker, and carefully tried out each of the configurations in Figs. 2.6, 2.7, and 2.8. She made sure that A didn’t just

nudge B, but collided with it. To their delight, they found that the results of the experiment were exactly as Zak had guessed. They couldn't wait to get to school to tell Ebin.

All three of them had arrived early that day, so they had a chance to talk before the school bell rang. When Zak and Anju told Ebin about their experiments with beads, balls and rods, Ebin raised a question.

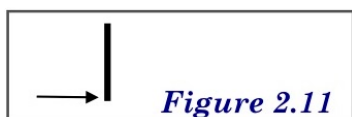
Ebin: Good insights. I wonder though. Instead of a straight rod and a striker, what would happen if A and B are both straight rods?

Anju: Hmm! We haven't tried that. Let me draw something and see how that goes. (Draws...) Okay, so this figure is somewhat parallel to Fig. 2.6, the first scenario with the sphere:



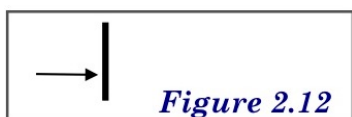
If A and B are both straight rods, and A hits B at the center, I think B would move in the same direction as A.

This figure is the counterpart of the second scenario, in Fig. 2.7:



If A were to hit B at one of the ends, B would rotate.

And this one is like the third scenario, in Fig. 2.8:



If A were to hit B at a point **between** the center and the edge, B would perhaps rotate and simultaneously move in the direction of A's motion.

Ebin: Wait a minute. Are you saying that if A hits B at an end, as in Fig. 2.11, B will rotate, but not move forward?

Anju: Well, my guess is, the end that gets hit would move forward, but the other end would remain where it is. So, the object as a whole would rotate around the end that doesn't move. The entire rod doesn't move forward, but the center would move as part of the rotation.

Ebin: I'm not convinced. My intuition is that the end that is not hit will also move, but in the opposite direction. The entire rod would rotate around some point on the rod, between the middle and the end that is not hit.

Zak: We'll have to do some experiments to test this. So let's hold it as unresolved for now.

Just then, the bell rang.

Ebin: Saved by the bell!

When they sat down to eat lunch, Ebin and Anju found that Zak had thought through the issue of a rod on the one hand and a sphere or disk on the other. He began in a professorial tone:

Zak: All this time, we have been talking about two kinds of motion. One is moving in a straight line. What they call *translation* in geometry, causing *displacement*. The other is *rotation*. Now, when a force acts on an object O, O is displaced from one location to another under certain conditions; it undergoes a *change of location*. Displacement is change of location. This is what happens to a tennis ball when a tennis racket hits it, or to a stone when you throw it. Rotation, on the other hand, is a *change of orientation*, right? So when an object rotates, there is a change in the direction in which it faces.

Ebin: Yes, I can see the change in direction when a rod rotates. But what about when it's a disk? A disk doesn't have a direction, does it?

Anju: Oh yes, it does, Ebin. If you draw a line through the center of the disk, you can see the change in its direction as it rotates.

Zak: That's right. So the conditions for rotation are different from those for displacement. Rotating is what happens when the wind turns the blades of a windmill, or flicking a switch moves the blades of a ceiling fan.

Anju: And a pinwheel, when the wind blows!

Zak: That's right. A pinwheel has rotation, but no displacement.

SOMETHING FUN TO DO

If you feel like making a paper pinwheel, you can find out from these videos how to do it.

<https://www.youtube.com/watch?v=muywzgSIaqq>

<https://www.youtube.com/watch?v=5KJPTb7Pg4o>

Ebin: Nice, displacement is change of location, and rotation is change of orientation. That sounds like a good way to distinguish them. We were talking the other day about using force to turn the tight lid of a bottle. That is using force to change its orientation without changing location. So under what conditions does something change its location, and under what conditions does it rotate?

Zak: That's exactly what we have to figure out.

STOP AND THINK 2.3

Ebin asks: "... under what conditions does something change its location, and under what conditions does it rotate?"

Can you answer Ebin's question?

State your reasons for your answer.

Anju: It's possible for a body to change location *and* orientation at the same time, isn't it? As in the case of the wheels of a bicycle you are riding on. So we also need to find out under what conditions it does both.

Zak: Absolutely! When you ride a bike, you're moving forward, changing location, and so is the bike. So there is displacement. The hubs of both the wheels are rotating. So there is change of orientation. What about the wheel itself? Is it rotating?

Anju: The entire wheel rotates, doesn't it?

Ebin: Are you wondering if it also revolves?

Anju: M-hmm. But what exactly *IS* revolution? And how is it different from rotation?

Ebin: Imagine an ant sitting on the edge of the bicycle wheel.

Anju: What about it?

Zak: I see what you're getting at, Ebin.

Ebin: Would the ant be rotating?

Anju: Of course not!

Zak: Would it be revolving?

Anju: Oh yes, I get it! Revolving around the hub.

Ebin: Then you know what? A white spot on the edge of a rotating wheel would also be revolving around the hub.

Zak: Exactly. Like when a merry-go-round rotates, its seats revolve around its hub.

Anju: Okay, so suppose my bicycle goes around a tree in a circular path, about ten feet from the tree. The wheels would be rotating. And the bicycle would be revolving around the tree, right? Just like the earth going around the sun.

Ebin: Great example of rotation *AND* revolution!

Anju: A twirling sphere whirling around a burning sphere in a curved path!

Ebin: Oh, the poet!

Anju: On a serious note, when the earth rotates and revolves around the sun, are WE revolving around the center of the earth? Are we rotating too?

Zak: After lunch!

After lunch, they continued the conversation.

Anju: I have a question. Suppose we place my bicycle upside down on a table. If we push on one of its pedals, the back wheel would move. It would rotate, but it wouldn't move forward. But if the bicycle wheels are on the ground and we push down on a pedal, the bicycle moves forward while the wheels rotate. My question is, how does the downward movement of the pedal cause the wheels to rotate, and how does the rotation of the wheels cause the forward motion of the bicycle?

There was silence. Everyone was contemplating. Zak was the first to speak.

Zak: To look for an answer to that question, Anju, I think we need to understand the relation between rotation and displacement.

Ebin: Hmm. And it looks like we'll need some time to figure this out.

STOP AND THINK 2.4

Can you answer Anju's question?

[You may first need to build a Theory of Bicycles.]

2.3 Identifying the Essentials

As soon as Ebin got home from school that afternoon, he ran to his mother, Lisa.

Ebin: Mom, there's something I want to talk to you about.

Lisa: I'm all ears, Ebin.

Ebin: Anju posed a question today. If we place a bicycle upside down with the wheels up in the air, and push on the pedals, the wheel would rotate, but the bicycle wouldn't move forward. But if the wheels are on the ground and we push down on the pedal, the bicycle moves forward while the wheels rotate. How does the movement of the pedals cause the wheels to rotate, and how does the rotation cause its forward movement?

Lisa: Hm! Good question.

Ebin: Zak thinks we need to first understand the relation between rotation and displacement. And to understand that relation, I want

to first simplify Anju's question a bit. What are the conditions under which displacement causes rotation? And under what conditions does rotation cause displacement?

Lisa: Sounds like you would need to construct a theory of bicycles.

Ebin: Oh yeah, exactly! Well, I'm going to focus on wheels and surfaces for now, and not worry about the pedals and chains.

Lisa: Yes. Focus on what is essential in what you are trying to figure out, and eliminate all the irrelevant details. That's a good strategy.

Ebin: Okay! A question. Can we take something simpler for this thought experiment, something that doesn't have a balance problem that a bicycle would have? Suppose we take a wooden board with edges A, B, C and D. It has four legs with wheels. (He drew a diagram on a piece of paper.) It looks like a cart, so let's call it a cart.

If the board is small enough, say, two feet by three feet, and we hit edge B with a cricket bat, the board will move forward, with side D in front, right? It will move in the same direction as the movement of the bat. The wheels would also be rotating.

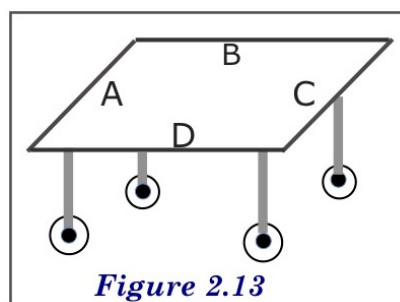


Figure 2.13

It wouldn't matter whether you hit the edge of the board, the leg, or the wheel. The result would be the same. There will be displacement of the entire object, and rotation of the wheels.

But if the cart is placed upside down on the floor, and you do the same thing, there might be displacement of the cart, but no rotation of the wheels. Why?

Lisa: Okay! Why?

Ebin: I think it has to do with the wheel being in contact with the floor. Suppose we place the cart on a carpet and push it. The pushing would cause the wheels to rotate. And the rotation would cause displacement of the cart. Now if we pull the carpet out from under the cart while we hold the cart, the carpet is displaced, but the cart doesn't move forward. But the wheels are in contact with the carpet, and so they move, resulting in rotation.

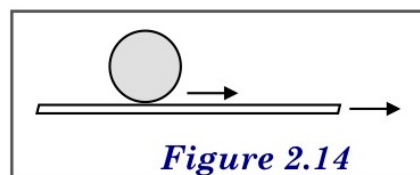


Figure 2.14

So we should conclude that it is the linear motion of the wheels relative to the carpet that makes the wheels rotate. They rotate

whether we push the cart on the carpet, or pull the carpet from under the wheels. Am I making sense?

Lisa: Yes, I can follow your reasoning.

Ebin: Good. What I just said, I'm going to try to state it as a principle:

PRINCIPLE OF ROTATION:

When a wheel is in contact with a surface, the relative linear motion of the wheel with respect to the surface causes the wheel to rotate.

Lisa: Good. You should write it down. Can you also write down the principle you wanted to talk to me about? I'd like to check it.

Ebin wrote both principles on a piece of paper:

PM-10a: *When a moving object X causes a stationary object Y to move, Y moves either in the same direction as the movement of X, or at an angle to that direction.*

PM-10b: *When a moving object X collides with the flat surface of a body T such that X bounces off T, the angle of incidence is equal to the angle of reflection.*

Lisa: Your principle doesn't say anything about the contact between A and B.

Ebin: Oh yeah! Let me add another principle:

PM-11: *For a moving object X to cause a stationary object Y to move, there must be contact between the two bodies.*

Lisa stared at the two principles, and was silent for a while.

Lisa: That might work. But I should tell you this, Ebin. You are referring to your statements as 'principles'. In Physics, these statements would be called 'laws'. Like "Newton's Laws of Motion." So perhaps you could call them laws.

Ebin: But laws are to be obeyed, right? Like traffic laws.

Lisa: In this context, 'law' means something else. It is the statement of a regularity in nature.

Ebin: Oh! So a law is the same as what we call a principle!

Lisa: Alright, I have to get to a meeting. Good luck with your laws. See you later.

Ebin was ecstatic. He called Anju and Zak, and reported to them his discussion with his mother, and told them about the new principle. He also told them about the idea of laws of nature.

Zak: Great!

2.4 Theory as a Configuration of Laws

The next day, Anju, Zak and Ebin had hurried through lunch and were at the whiteboard in the classroom. Ebin spoke excitedly, but was somewhat disorganized. So Zak took charge. He went to the whiteboard, and got ready to write.

Zak: We now have a theory of motion with several connected principles, or I should say, laws. We need to put all the laws together in one place. We should write down the observational generalisations (OGs) we want to explain. First let's write the laws that explain them.

Zak wrote, with Ebin and Anju reading out the laws from their notes.

A THEORY OF MOTION OF INANIMATE OBJECTS	
Law 1:	<i>Force causes change: of location, or of orientation.</i>
Law 2:	<i>Inertia is the resistance to change. All objects have inertia.</i>
Law 3:	<i>The greater the weight of an object, the greater its inertia.</i>
Law 4:	<i>Moving objects have force.</i>
Law 5:	<i>When a moving object X collides with a stationary object Y, the force of X causes Y to move.</i>
Law 6:	<i>The greater the inertia of an object, the greater the force needed to make it change.</i>
Law 7:	<i>The greater the weight of a moving object, the greater the force with which it acts on a stationary object.</i>
Law 8:	<i>The greater the speed of a moving object, the greater the force with which it acts on a stationary object.</i>
Law 9:	<i>The greater the force acting on a body, the faster it changes its location or orientation.</i>
Law 10a:	<i>When a moving object X causes a stationary object Y to move, Y moves either in the same direction as the movement of X, or at an angle to that direction.</i>
Law 10b:	<i>When a moving object X collides with a flat surface T such that X bounces off T, the angle of incidence is equal to the angle of reflection.</i>
Law 11:	<i>For a moving object X to cause a stationary object Y to move, there must be contact between the two bodies.</i>

Zak: Okay, so now let's list all the phenomena that we need to explain.

When they were done, this is what they had on the board:

PHENOMENA: WHAT OUR THEORY SEEKS TO EXPLAIN

- OG-A: When a moving carrom striker hits a carrom piece, the piece moves.
- OG-B: When a moving rod A hits the center of another rod B perpendicular to it, B moves in the same direction as A. If A hits B at an end, B rotates. (We still have to do experiments to figure out where the center of rotation would be.)
- OG-C: If a cricket bat hits a not-so-heavy cart on wheels, the cart moves in the same direction as the bat, and the wheels rotate. But if we turn the cart upside down, and hit it with the bat, the cart moves in the same direction as the bat, but the wheels do not rotate.
- OG-D: If we hit the cart with greater force, it moves faster.
- OG-E: If a moving ball A hits a stationary ball B, and B moves, the greater the speed of A or the greater the weight of A, the greater the speed of B.

Ebin: (disappointed) We explain five things with so many laws?

Anju: Don't worry, Ebin. As we develop our theory further, we'll find more and more things to explain. And we may not need all the laws, so the laws will be fewer. At least, that is what we need to try to do.

CHAPTER 3

SEESAWS AND LEVERS

3.1 The Behaviour of Seesaws

After his usual Sunday morning run in the park, Zak looked for a place to sit down to catch his breath. The seesaw was perfect. He sat down on one side of the seesaw, munching peanuts. Before long, he was sitting with eyes closed, lost in thoughts about the homework for the next day.

Suddenly, he felt his side of the seesaw go up.

Zak: [muttering to himself] How on earth did that happen?

Anju: [hearing him mumble] How on earth did *WHAT* happen?

Zak opened his eyes, and saw Anju sitting on the other end of the seesaw, grinning at him. Her side had gone down. When did she come? He hadn't noticed her.

Zak: When you sat on the seesaw, your end came down! Don't you see...?

Anju: See what? Should it have gone up into the sky?

Zak: No, silly, what I mean is... [pausing hesitantly] Look, suppose you were sitting on one side of the seesaw, with no one on the other side. You would expect your side to be down, touching the ground, right? But what if I came and sat at the other end? My end would go down and yours would go up.

Anju: Right.

Zak: That's what we expect, because I'm heavier than you. But when you sat down at the other end, your side went down! That's what surprised me. It was the exact opposite of what we expect. Right now, your end is touching the ground, and my end is up though I'm heavier. How did that happen? How do you explain this?

Anju: Huh! How *DID* that happen? [Pauses] You know, Zak, I have a feeling that the seesaw is opening the door to more laws.

Zak: You have something specific in mind?

Anju: Yep! Suppose there's a law like this:

LAW OF SEESAWS

If A and B are sitting on a seesaw, one at each end, and if A is heavier than B, then A's end would be down and B's end would be up.

Zak: That's pretty good. But hold on! What we are seeing seems to go against it! I am heavier than you, but my side went up!

Anju: But that is because the see saw is not fair. The law is not wrong, but it applies only to seesaws that are fair.

Zak: What's a fair seesaw? Are you saying that among seesaws, there are fair ones and unfair ones?

Anju: Yep. Like this one we're sitting on is unfair. That's why my side went down. So we have to revise the law:

LAW OF SEESAWS REVISED

If A and B are sitting on a fair seesaw, one at each end, and if A is heavier than B, then A's end would be down and B's end would be up.

Zak: Hey! You can't do that! It would be just like saying, "People with a pure heart can walk on water." You can't test this statement, because if someone can't walk on water, you would simply say that the person doesn't have a pure heart. Unless you define 'pure heart', this law is not testable.

Anju: I see what you are saying. But how is that relevant for fair seesaws?

Zak: What exactly do you mean by 'fair seesaw'? How would we know if a seesaw is fair? Every time you come across a counterexample, you would say, "Oh, that's because it's not a fair seesaw." Then you can always claim that your law is true. It can never be tested and shown to be false — we can never *falsify* it. To test the law, we have to say what we mean by 'fair seesaw', and what would not count as a fair seesaw. For this, we have to *define the concept*. Without that, we can't figure out the conditions under which the side with the heavier person would be down, or up.

Anju: What you are saying is, a law should tell what kinds of counterexamples would show that the law is not true.

Zak: Exactly.

STOP AND THINK 3.1

State a 'Law of Seesaws' without using the idea of 'fair seesaw'.

3.2 The Behaviour of Levers

On Monday morning at school, Ebin listened attentively as Anju and Zak talked about their experience at the park — the puzzlement of why Anju’s side of the seesaw went down, and Zak’s side went up, even though Anju was lighter.

Ebin: Listen! It will be much easier if we have pictures. So let’s start with a picture of a seesaw, something like this. (Talks as he draws Fig. 3.1.)

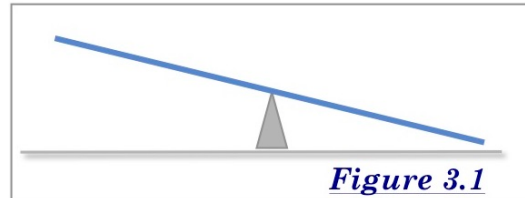


Figure 3.1

What I have done here is to create an abstraction, removing everything that is not relevant for our theory. The only things that are relevant are the length of the seesaw, and the pointed shape in the middle, which supports it. The breadth of the seesaw, whether it is made of wood or metal — they are not relevant.

Zak: That’s a good start.

Anju: Talk to me about the picture, Ebin!

Ebin: Well, you have the fulcrum, the triangle F. The seesaw plank sits on the fulcrum, with the fulcrum at the center, right? The plank has two sides, which oscillate, going up and down. Let’s call the sides A and B. And their movement is rotatory. (Adds labels to get Fig. 3.2.)

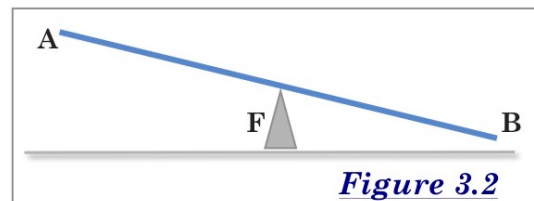


Figure 3.2

Now let us try this. Suppose Anju sits on one side of the seesaw, close to the fulcrum. (Adds an oval: Fig. 3.3.) That is Anju. Anju’s weight will make Side B go down, right?

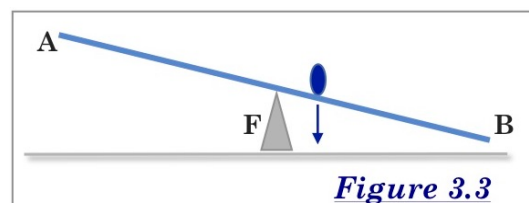


Figure 3.3

Suppose we want to make Side B go up so that the seesaw is horizontal. We can apply force at the end (where the arrow is) by either pulling or pushing it up. (Fig. 3.4.) How much force would we need to use?

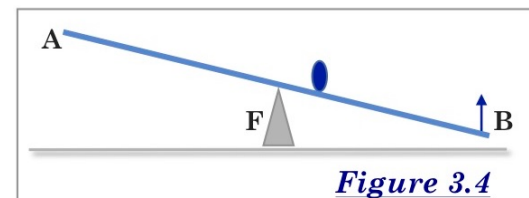
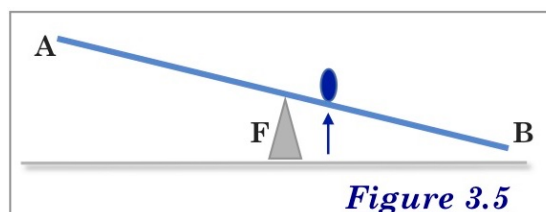


Figure 3.4

Anju: Isn’t it just like when you want to open a heavy door? If you push it close to the side with the hinges, you have to push really hard. But if you push it on the other side, you need to use a lot less effort.

Ebin: And what if I was trying to push Side B up at the point where you are sitting, Anju? (Fig. 3.5)



Zak: That would need much more force. More than you would need if you push up at the end. In fact, the force will have to be equal to Anju's weight for the side to move up.

Ebin: And what if you, Zak, are the one sitting on the seesaw?

Anju: He weighs more than me, so you need even more force to push him up.

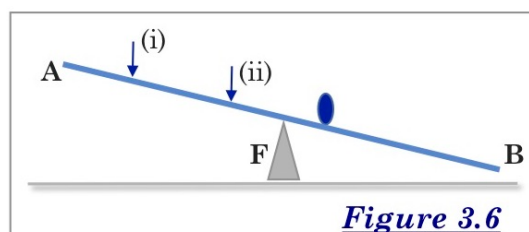
Zak: Interesting. Let us change the scenario a bit. Anju is sitting on Side B, exactly as in the earlier case, close to the fulcrum. Instead of pushing side B upward, we could make Side B go up by pushing Side A downward, at different points.

Let's consider two of them.

One would be close to the end of A. How much force would you need to apply?

And the other point for pushing down on Side A is at the same distance from the fulcrum as Anju on Side B. How much force would you need to

apply in this case?



Just then, the bell rang for classes to begin, and the three of them had to postpone the discussion till later. After school, at Ebin's suggestion, they walked to his home to continue their exploration.

Once home, Zak plunged into the conversation right away.

Zak: Let us go back to our earlier question. In the scenario in Fig. 3.6, how much downward force would we need to apply at each of the points, (i) and (ii), to make Side B go up?

Ebin: Whether we are pushing side A downward or side B upward, the force needed would be the same, right?

Zak: Yes! There *HAS* to be a pattern here. Can we come up with laws for the behavior of seesaws? What are the factors we need to take into account?

Just then, Ebin's mother Lisa walked into the room.

Lisa: Hello, Anju, Zak, Ebin! Just back from school and already into discussion, eh? I heard a bit of it. Zak, you were talking about the

factors you would need to take into account to figure out something. I think what you had in mind was, what are the relevant ‘variables’ you need to take into account.

Anju: What do you mean by ‘variable’?

Lisa: Oh, it’s just a word that you might find useful later. Zak actually gave an explanation for why his side went up even though his weight is greater. The distance between the person on the seesaw and the center — point F — can vary, they can be different, so that is a variable. The weight of the two people can also vary, so that is also a variable. A VARIABLE is something that varies.

Anju: Can you give some other examples?

Lisa: Let me try. All adult humans have a brain, and a heart. This doesn’t vary from human to human. But humans also have properties that do vary. There are differences in height. Also in hair colour, skin colour, and eye colour. So among the properties of humans, height, hair colour, skin colour, and eye colour are variables. These variables have VALUES. A person’s eyes could be black, dark brown, light brown, blue, or green. When we specify the actual colour of someone’s eyes, we are specifying the *value* of that *variable* for that person.

Anju: Ah, so you are you saying that there are variables in the behaviour of seesaws?

Lisa: Exactly. One variable that affects the behaviour of a seesaw has to be the weight of the object on it. Another variable might be the distance of the object from the fulcrum. Once you have identified a possible variable, you have to test it, perhaps experimentally. You haven’t been doing that, have you? It’s important!

Ebin: Yes, Mom! We should have been doing some experiments to figure things out. We’ll do them.

Zak: Hang on! What if we start with one half of a seesaw? Make the other side dotted, and ignore it for now. [Draws Fig. 3.7.] The thick line is for Side B, the triangle for the fulcrum, and an oval object placed on Side B. What we’re trying to figure out is: How much force is needed to raise Side B such that the seesaw is horizontal? We know how much force is needed if we apply it right below the weight: the force would be the same as the weight of the object pushing downward. And we agreed that if the force is applied at the end of the side, the effort would be much less.

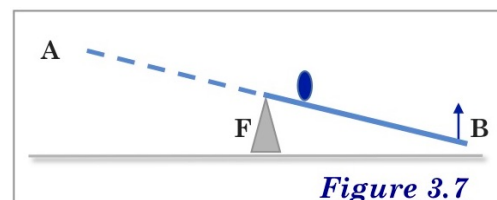


Figure 3.7

Ebin: You know what you just did, Zak? You've converted the seesaw into a lever!

Anju: That's right! A seesaw is actually a kind of lever, isn't it? So the laws for a seesaw would be the same as the laws of a lever.

Ebin: What are the relevant variables for the laws then?

Anju: Hmm, given an object on the seesaw, the force needed would depend on its distance from the fulcrum. That would be one variable. Another would be the distance from the fulcrum to where the force is applied.

Ebin: Nice, Anju!

Zak: That's fine, but what about the weight of the object?

Ebin: Of course that would also be a variable. How about this for a law?

LAW 12: *For an object placed on a lever, the greater its distance from the fulcrum, the greater the force required for it to move up.*

Anju: Here's another law then:

LAW 13: *For an object placed on a lever, the greater the distance from the fulcrum to where the force is applied, the lesser the force required for it to move up.*

Zak: And another one:

LAW 14: *The greater the weight of the object, the greater the force required for it to move up.*

Anju: Wait. Like we said, we can lift Side B by pushing down on Side A as well. So let's bring back Side A! Then it would be exactly as in the diagram we had earlier. This one.

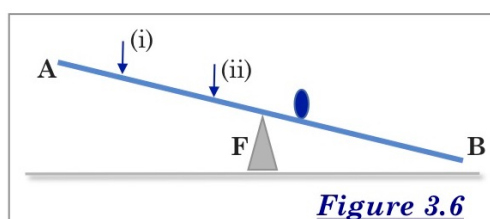


Figure 3.6

Ebin: Yes, let's take the whole seesaw. It doesn't matter if we are pushing down side A or pulling up side B. The same laws would apply, right?

Zak: Yes! Suppose we also place an object on Side A. Let me draw that. [Draws Fig. 3.8] We now have weights on both sides of the seesaw.

Anju: Let's say that the blue object is heavier than the orange one. The question is, under what conditions would placing the orange object on Side A make Side B with the blue object go up?

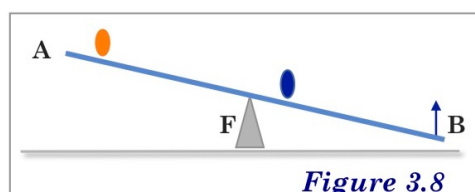


Figure 3.8

Zak: Aha! This is where we started, right? Why Anju's side went down and mine went up. How about the laws we just stated, would it work?

Law 12: *The greater the distance between the fulcrum and the object, the greater the force required for lifting the object.*

Law 13: *The greater the distance between the fulcrum and the location of the force applied, the lesser the force required for lifting the object.*

Law 14: *The greater the weight of the object, the greater the force required for lifting it.*

Ebin: We're back to the laws we already have! And we're back to the question this all started with, when Zak, your side was up and Anju's was down. So the question we've clarified is:

Under what conditions would the side with the heavier object be up and the side with the lighter object be down?

Zak: Wait a minute. If there are objects on both sides of the seesaw, their weights are forces pushing downward. So we can ask:

Under what conditions would the forces on the two sides be equal?

Anju: Okay, this is the first law that we came up with:

Law 12: *The greater the distance between the fulcrum and the object, the greater the force required for lifting the object.*

Let us say that the distance between the fulcrum and the object is D , and the force required for lifting the object is F . So what we are saying is that the value of D is *proportional* to the value of F .

Ebin: Value of F ?

Anju: Yes, we can measure the distance using a meter rod. The readings on the meter rod would be the value of the variable D . Suppose we had some instrument to measure how much force we need to apply to make the heavy object move up or down. That is what I mean by 'value of F ', just like there is a 'value of D '.

Ebin: Ah, I see. So we will have to find a way of measuring F .

Anju: Yes. But if there is a way of doing that, we can state the law as:

The value of F is proportional to the value of D .

Ebin: Oh, don't we use a symbol that looks like the symbol for alpha to say, "is proportional to"? So we can use that symbol and write the law as:

$$F \propto D$$

Zak: Cool. Now, what about the second law? It talks about the distance between the fulcrum and the location of the force, not the location of the object.

Law 13: *The greater the distance between the fulcrum and the location of the force applied, the lesser the force required for lifting the object*

So if the distance between the fulcrum and the location is D^2 , we can say:

F is indirectly proportional to D^2 .

Shall we use the alpha symbol like this?

$$F \propto 1/D^2$$

We have to do some experiments to test these laws of direct and indirect proportionality of the force and distances. But till then, let us assume that these laws work.

Ebin: Neat. Let us do that. What about the third equation?

Just then, the bell rang, and the young researchers ran to their classrooms.

Ebin: Great! Let's...

Anju: My head is smoking. I can't do this anymore. Can we come back to it tomorrow?

Ebin: Sure, let's do that.

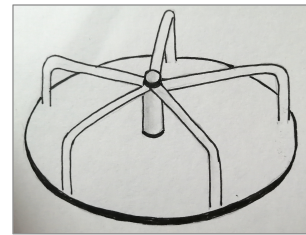
[This conversation continues for a bit, and is available in Appendix 6: "From Verbal Statements to Equations," a transition from words to formal symbols.]

CHAPTER 4

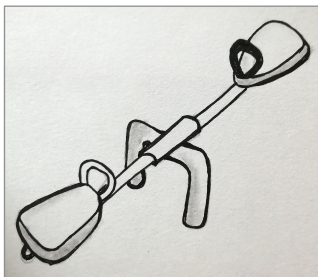
SLIDES AND INCLINED PLANES

4.1 Slides and Swings

One morning, Ebin walked over to the playground next to his home, and sat down on a swing. He was looking around when his eyes fell on a circular disk with bars, which were handles to hold on to when riding on the disk, or for someone who was not on the disk to make it spin by pushing or pulling on the handles.



“Some people call it a merry-go-round, and others call it a roundabout,” Ebin thought to himself. “Whatever it is called, it behaves just like a bicycle wheel placed horizontal. A theory of the motion of roundabouts or merry-go-rounds would be the same as a theory of the motion of wheels!”

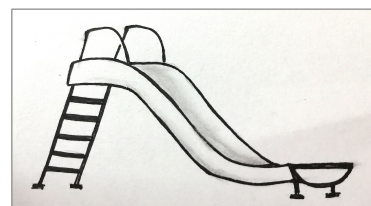


He looked around the playground again. It had a couple of seesaws. He and his friends Anju and Zak had just been trying to develop a theory of seesaws as levers. He also saw slides. And swings, like the one he was sitting on. How about developing a theory of slides and of swings?

Like Zak and Anju, Ebin had a constantly active mind, bubbling with new ideas, thoughts, and questions — when out for a walk, brushing his teeth, taking a shower, lying awake in bed, or just as he was about to fall asleep. Engaging with those ideas and questions had helped all of them learn a great deal, both individually and together. For them, learning was not something that happened only at school, or from teachers. They learnt a lot when they were engaging with what interested them, and when they were doing something they found joyful.

Smiling to himself, Ebin watched the children playing there, and thought: *A playground has so many things for the body to play on and learn. It also has lots of things for the mind to play with!*

He saw a child go up the steps to the top of a slide, sit down, and lean forward. And she went sliding down, laughing.

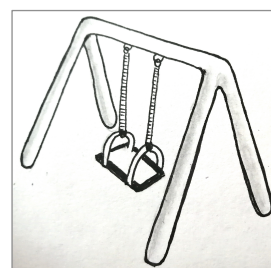


Ebin said to himself: *If the slide was horizontal, there would be no motion. And if the slide was vertical, the child would simply fall straight down. So the sliding motion cannot happen vertically. It can happen only if the sliding surface is at an angle, so that an object can come down by moving forward and downward on it. This must be because the force of gravity makes it move. And gravity is a downward vertical force. That would explain why something on a horizontal slide doesn't move.*

What about how fast the child slides down? I'm sure the speed depends on the angle of the slide, Ebin found himself saying aloud. This would be like riding on a bicycle down a hill. The bicycle moves without pedaling. And the steeper the slope, the faster the motion.

Is that the only law needed to explain that downward motion? he wondered.

Just then, the swings for little children caught his attention. Children sat on the swings, and their parents pushed them from behind, setting each swing in motion. When a swing slowed down, the parent gave it another push.



Ebin muttered to himself: *The speed of the swing depends on how hard it is pushed.... But wait a minute! When you push a swing,*

isn't it like a rod that goes back and forth (rotates) when one end is fixed to something and the other end is free?

This was something that he had tried to work out with Zak and Anju while exploring the motion of a bicycle wheel. His thoughts came crowding now: *There's an important difference between a bicycle wheel and a swing. If you apply force at the edge of a bicycle wheel in a direction that is perpendicular to its radius, the wheel rotates. In the case of a swing, instead of pushing it, you could pull it back and let go, and it would still move forward. As it moves, it goes faster, then slows down, and stops. And then it swings back.*

What makes it move in this particular way?

He couldn't think of an answer right away, but he was really excited. He rushed home to discuss it with his mother. But when he got there, she was nowhere to be found. Then he noticed a note on the fridge. It said, "Ebin, I am going out to visit a friend. Have left some snacks for you on the table."

He was disappointed; he wanted to talk to her about slides and swings. So he did the next best thing; he called Zak.

Ebin: Can you and Anju come over? There's something we should talk about — to find laws to explain something that I found puzzling.

Zak: Sure, we'll be there in half an hour.

4.2 A Dropped Stone Moves Downwards. Why?

Ebin opened the door when he heard the noise of their creaky gate. Anju and Zak walked in and got comfortably seated on Ebin's bed. Ebin gave them a quick summary of what he was excited about — the thoughts that the slides and swings had triggered in his mind.

They were all silent for a while, inside their own thoughts, till Anju spoke.

Anju: Remember the very first law we formulated? We said that
Inanimate entities that are not manmade move only when an external force causes them to move.

Aren't both slides and swings manmade?

Zak: So?

Anju: When I sit on top of a slide and lean forward, giving myself a little push, I slide down, right? But when I sit at the bottom of the slide, I don't slide up. The same is true of inanimate objects, whether a stone, or a tennis ball. So what makes an inanimate object slide down?

Ebin: Gravity, obviously!

Anju: Do you mean gravity is a force?

Ebin: M-hmm. The external force that causes downward motion.

Anju: Where does it come from?

Ebin: From within the Earth, of course.

They fell silent again. Ebin got up and began walking back and forth, his hands clasped behind his back.

Zak: Ebin is right. We know that if we hold a stone up in front of us, and let go of it, it moves down in a straight line and lands on the ground. The same way, a stone on a slide only moves downward, not upward or sideways. Suppose we generalize it as a law:

Law 15: The earth has the property of gravity, a force that causes any object with weight to move towards it in a straight line.

Anju: Hmm. Stones that are dropped and stones on a slide have something in common: they both move downwards. Neither of them moves upwards, or sideways.

Ebin: Hang on! If we drop a stone, it moves vertically downwards. But a stone on a slide moves at an angle. Don't we have to explain why? Gravity causes downward vertical motion. How does that get translated into motion that is neither vertical nor horizontal, but at an angle?

Anju: If there was only gravity, the motion would be vertically downward. But there's the slide stopping it. So it moves downward along the surface. Since the surface of the slide is an incline, the motion is also inclined. There is no other way for it to go down. To come down vertically, the stone would have to pierce through the slide, making a hole.

Ebin: Nice.

Zak: Nice indeed. We will need to reformulate our Law 15 to accommodate coming down slopes.

4.3 Action at a Distance

Zak: Okay, that is taken care of.

Anju: But there's another problem.

Ebin: Uh-oh! What is it?

Anju: Our Law 11 says:

Law 11: For a moving object X to cause a stationary object Y to move, there must be contact between the two bodies.

Now, gravitational force doesn't demand that the bodies be in contact. A stone could be kilometers away from the earth but still moves towards the earth in a straight line. So the earth attracts the stone without being in contact with it.

Ebin: (after quickly checking the Internet) That is true. The Sun's gravitational force on the earth holds across more than 14 million kilometers.

Zak: Right. That's not action through physical contact. It's action at a distance, unlike what happens in collision.

Anju: Okay, so that means we have to revise Law 11.

But Ebin pointed out something important.

Ebin: Look, our law says: "For a *moving object* to cause a *stationary object* to move, the two objects must be in contact." We have not said: "For an object to cause another object to move, the two must be in contact." This means that according to the law, contact is required only when something starts moving because of a collision. The stone's movement towards the earth is not caused by a collision. So we don't need to revise Law 11. We just have to bear in mind that gravity is "action at a distance."

Ebin could see that Anju and Zak were relieved. Now that they had formulated a law for slides, they decided to get to the behavior of swings.

4.4 Two Competing Theories of Motion

Anju: There's something bothering me.

Zak: What?

Anju: Remember we formulated our Law 2 like this?

Law 2: Inertia is the resistance to change. All objects have inertia.

What did we have in mind when we said 'resistance to change'? Were we talking of just change from rest to motion? That would be a *change of location*. Or could it be other kinds of change? For example, *change of speed*. If we take rest as being zero speed, change from 'rest to motion' would mean both change of location as well as change of speed, right?

Zak: I only had change of location in mind. But you're right. When a moving ball hits a stationary ball, the second one changes from rest to motion, which also involves change in speed, from zero.

Anju: But what about the opposite case, decrease in speed? Change from motion to rest would be change from non-zero speed to zero speed.

Zak: You're right.

Anju: Okay, so if we throw a stone up, it first changes from rest to motion, but then it slows down, and stops for just a moment — a fragment of time we may not even be able to measure. Then it starts coming down. There is also a change in speed — it comes down faster. Do these different kinds of change of speed need some kind of force?

Ebin: Yes, of course! That decrease in speed as the stone goes up, ending in a state of rest, and the increase in speed as it comes down, are both caused by gravity.

Anju: Why do we need gravity for that? Couldn't it be like this? When we throw a stone up, we transfer force from our hand to the stone. And when that force in the stone gets used up, the stone comes to rest.

Zak: That's interesting! A game of force and inertia!

Ebin: We now have two different ideas of force and inertia related to motion. Should we write them down so that we are clear about them?

Anju: Okay, let me try. I'll call them Idea A and Idea B. I'll need help. We'll have to keep in mind all the laws we have stated so far.

Ebin: Go ahead and write.

Anju wrote, with prompting from Ebin and Zak, and with many revisions along the way.

Idea A: Change of location (motion) requires force.

FORCE causes motion. An object X in motion has force in it. When X collides with an object Y at rest, that force gets transferred to Y, and Y changes from the state of rest to motion. As Y moves, the force gets used up, and when all the force is gone, Y comes to a state of rest.

INERTIA is the state of rest.

Idea B: Change of speed requires force.

FORCE causes change of speed. The change could be an increase, including change from rest (zero speed) to motion (non-zero speed), or a decrease, including change from motion (non-zero speed) to rest (zero speed). When an object is moving at constant speed, there is no force acting on it.

When an object in motion collides with another object, it causes a change of speed in that object.

INERTIA is the state of constant speed, including the state of rest.

Ebin: You know what? These are two distinct theories of motion!

Zak: Actually, I would say that they are two distinct *ideas* for theories of motion. Just starting points. Or theory-seeds. To become theories, we have to develop them with clear definitions and laws, and perhaps more ideas.

Ebin: Perfect! Where did you get the idea of seeds, Zak?

Zak: I read it somewhere and it stuck in my head. And if I understand it, this is a good example of the seed idea.

Ebin: Great! The question is: How do we choose between these two theory-seeds, which are in competition? To make that choice, we will have to develop both of them into full-fledged theories.

4.5 Force Causing Change of Speed

Ebin: Let's take Idea B first: what force does is cause change of speed. So, unless there is an external force, an object would be either at rest, or it would move at constant speed. Right?

Zak: Yes, exactly. Shall we use that to revise some of our earlier laws?

Anju: Like what?

Zak: Here, let me put the original laws and their revised versions side by side.

Law 2 (orig): *Inertia is the resistance to change. All objects have inertia.*

Law 2 (rev): *All objects have inertia. In the context of motion, inertia is the resistance to change of speed.*

Anju: That makes sense!

Zak: Let's do Law 5:

Law 5 (orig): *When a moving object X collides with a stationary object Y, the force of X causes Y to move.*

Law 5 (rev): *When a moving object X collides with another object Y, the collision causes Y to change its speed.*

Anju: Cool. We need to revise Law 6 as well. I don't know how to do that, but let me try, and tell me if it will work. Our earlier formulation had to do with overcoming inertia to move.

Law 6 (orig): *The greater the inertia of an object, the greater the force needed to make it change.*

Law 6 (rev): *For an object to change its speed, the degree of force applied on it has to be greater than its inertia.*

Zak: I think it will work.

Anju: Yippie!!

Ebin: Hold on, Anju, don't get so excited. We still haven't said anything about how to choose between the two ideas.

Anju: Oh! Can we do that later? We've done enough for a day, haven't we?

STOP AND THINK 4.1

Go through this Chapter and all the previous ones, and make a list of the final formulations of all the laws that we need in order to explain all the phenomena having to do with motion discussed in these chapters. Write them down to make sure that you have included everything that is necessary, and nothing that is unnecessary.

As Ebin points out, the young scientists have not found a way to choose between Idea A and Idea B. Would you like to try to do it? This will be a hard task, but it is worth a try.

STOP AND THINK 4.2

Watch this YouTube video, “Acceleration down an inclined plane,” at https://www.youtube.com/watch?v=2q_qzKT74Ow

It might help you get a better idea of how inclined planes work.

CHAPTER 5

SWINGS AND PENDULUMS

5.1 The Behaviour of Swings

The next day, when Anju, Zak and Ebin met in school, Ebin returned to the discussion they were having the previous day.

Ebin: Listen, we've talked about a seesaw being like a lever, and a slide being an inclined plane. How about swings? Isn't the behavior of a swing just like that of a pendulum?

Zak: Yes, we talked about this earlier, but didn't go into details.

Ebin: You know, we can think of a swing as being made of a rod or a chain that is attached to a fixed point on top, with a seat at the other end, to sit on.
[Draws in his notebook]

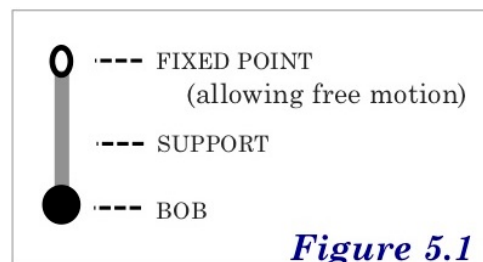


Figure 5.1

Anju: Ebin! You've drawn a pendulum with a bob! You did say, of course, that a swing with a seat is like a pendulum with a bob.

Ebin: Exactly. A swing and a pendulum both have this *abstract structure*. When we push a swing horizontally, the beam makes a rotating motion in one direction, and then in the other direction. Exactly as with a pendulum. And the seat or the bob moves along an arc. It can't move in a straight line.

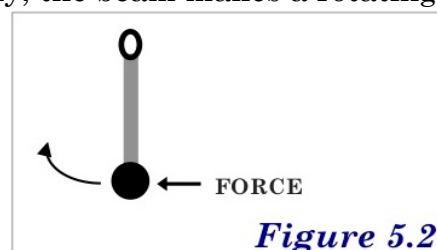


Figure 5.2

Anju: Our laws of motion should also explain the motion of the pendulum. Yes, the bob of the pendulum swings along a curved path. Now, when it comes down from one side to the midpoint, the question is, how does it go up the other side? Isn't that against gravity?

Zak: Yes, Anju. Why does the pendulum go up and down? We can say that it comes down because of gravity, just as in the case of the slide. But once it reaches the lowest point, why does it swing upward in the opposite direction? Going up doesn't happen with a slide. This is the question you raised, right, Anju?

Anju: Yep

Zak: That is one question we need to ask. There is also another one. The swing of the pendulum has two kinds of motion, a vertical one and a horizontal one. So the other question is:

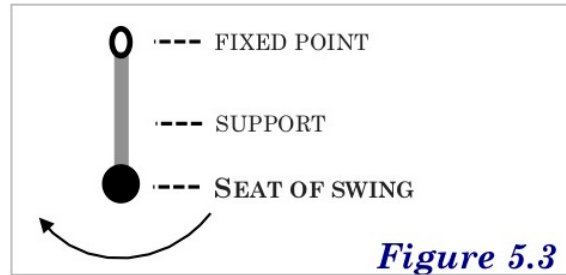


Figure 5.3

Why does the pendulum swing back and forth, from side to side?

Ebin: You know, instead of pushing, suppose we pull the pendulum to a side and just let go, the way we would let go of a stone from a height, and it drops to the ground. The pendulum goes through exactly the same back-and-forth and up-and-down motion. In this case, where there is no pushing or pulling, we know that the only force acting on it is gravity, the force that causes it to move downwards. So our question can be summarized as: How does that force cause the bob of the pendulum to move back and forth and up and down, again and again?

Zak: Hmm! It's something we'll have to sleep on.

Ebin: Hey, don't leave it just for sleeping on. Walk on it, eat on it, lie awake in bed on it!

Anju: Oh boy! I can feel my brain smoking.

5.2 A Pendulum's Amplitude and Speed

They continued their exploration the next day at Ebin's home.

Anju: I have a question. When the bob of a pendulum is set in motion, its speed gradually increases till it reaches the midpoint of its path. Then the speed starts decreasing, and the bob reaches zero speed at the other end. Then it reverses its direction and returns to the starting point. Again, on its return path, its speed gradually increases till it reaches the midpoint, and then it slows down. There must be a law on the speed of the pendulum.

Zak: Its motion is governed by gravity, right? So I would guess that the maximum speed depends on how long gravity acts on it, and also, how strong the force of gravity is. Can we say something like this? [Draws.]

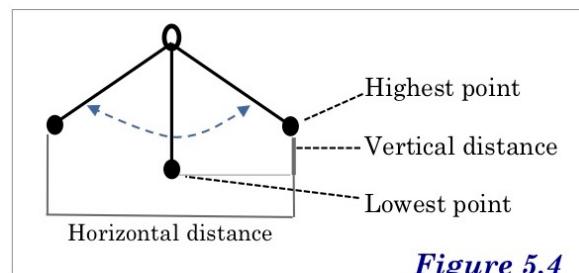


Figure 5.4

Anju: We can state the law as:

Law: The greater the vertical distance between the highest point and the lowest point in the swing of a pendulum, the greater the speed at the lowest point.

Ebin: That sounds kind of right. I was thinking the same thing, but didn't know how to formulate it. If we're right, it means that if you pull the bob away from the resting position, the farther you pull it, the greater the speed at the lowest point. There is this video that my mom showed me, where the distance between the highest and lowest points was called *amplitude*. Using that, we can also state the law as:

Law 16: The greater the amplitude of a pendulum, the greater the speed at the midpoint of its path of oscillation.

Zak: Hm! Nice. That's a clean statement of the relation between the amplitude of the pendulum and its highest speed. .

Ebin: There is a systematic relation between them, so that if we know the value of one of them, we can figure out the value of the other. In that video, they called that a *correlation*.

Anju: How do we find out if the law is correct?

Ebin: Not difficult, I suspect. Take a few pendulums, set them in motion, and observe their speed at the midpoint. If the speed decreases when the amplitude decreases, that confirms the prediction. If not, the law is false.

Zak: But it is going to be next to impossible to measure the speed of a bob when it is exactly at the midpoint.

Ebin: That just means we have to spend some time to figure out a good experiment design.

TASK 5.1

The claim is that there is a correlation between the amplitude of a pendulum and its speed. Can you design an experiment to test if the claim is true?

5.3 Designing the Experiment

Anju: As Zak said, it's impossible to measure the speed of a bob when it is exactly at the mid-point. I remember our teacher saying in Geometry class that a point has zero length and zero breadth. If the

mid-point is really a point in space and in time, how can there be any speed at that point?

Lisa was sitting with her laptop at the desk in a corner of the room, listening to the children's conversation. She rises now to join them.

Lisa: Ha, ha, now you are being a Zeno, Anju!

Anju: Huh? What?

Lisa: There was this Ancient Greek philosopher called Zeno. He argued that an arrow leaving a bow doesn't move. His argument was based on precisely the point you were making just now. Take any point in the path of the arrow. Given that a point has no length, the distance travelled by the arrow is zero. So the arrow stands still, it has no motion. And this is true at any arbitrary point on that path. *Ergo*, the arrow does not move.

Anju: What do you mean by *ergo*?

Lisa: It's a fancy Latin word for *therefore*.

Zak: And that's a fancy argument! But there must be something wrong with it, because the arrow does move, doesn't it? It changes its location, so it must move.

Ebin did a quick Internet search, and found a short 90-second video, "The Arrow Paradox," at:

<https://www.youtube.com/watch?v=cw3V5BruAwI>

And Anju found a 5-minute video by Professor Angie Hobbs, at:

<https://www.youtube.com/watch?v=IPNttsu8x24>.

Though only five minutes long, that video gave Anju a headache. So Ebin watched it with her, and tried to explain.

Ebin: If we think of time as a line, and if that line is made up of points that have zero length, as in Euclid's geometry, then Zeno's argument might be valid. What if we think of time as being made up of really short stretches? Then the arrow does move a tiny distance in a really wee bit of time. And if we think of it that way, then, at the mid-point, there is indeed a very short stretch of time where the pendulum moves fastest.

Zak: We still have a problem. If that mid-point is an extremely short stretch, measuring the speed accurately would be impossible.

They were silent for a while. Then Anju spoke.

Anju: How about this? Instead of formulating the law in terms of the highest speed for a given amplitude, what if we formulate it in terms of the average speed for that amplitude?

Ebin: Good idea.

Zak: Hang on, I don't get it.

Ebin: Let me try. Take the swing of a given pendulum from one end to the other. Suppose it begins at point X, and ends at point Y. We can measure the distance from X to Y. That's the **amplitude**. Call it **A**. And we measure the **time** taken for the pendulum to move from A to B. Call it **T**. The average *speed* of the pendulum from X to Y, then, is the total distance D divided by the total time taken, T. So our hypothesis is (writes):

As A (the **amplitude**) increases,
D/T (the **average speed**) also increases.

Does that make sense? If it does, then we don't have to worry about measuring the speed at the mid-point.

Anju: That's awesome! To test the hypothesis, then, all that we have to do is to measure D/T for different amplitudes, and check if the average speed increases as the amplitude increases.

Zak: But it is still going to be difficult to measure the amplitude or the time for each swing. If T is slightly more or slightly less than a second, we can't measure it even with a stopwatch.

They fell silent again, looking for a solution to the problem.

Ebin: Okay, here is what we can do. We measure the time taken for twenty complete cycles of the bob, and divide it by 20. This would give us the average time for each cycle. That way, we may be able to get a measurement where the error is very small, hence negligible.

Zak: And how do we measure amplitude?

Anju: We don't have to measure it exactly. Place a ruler horizontally, with its middle point aligned with the bob at rest. We let go of the pendulum at different points on the ruler. The more you move the bob away from the center, the greater its amplitude. So all that you need to do is measure how far you have pulled the pendulum to a side when you let it go. If we pull it away from its resting point by 4 cms in one case, and by 2 cms in another, the amplitude of the first one should be higher than that of the second one. So, suppose we swing it from different distances, 5 times from each distance. That way, we have five groups of 'swings' of the pendulum, each group with its own amplitude:

Swing Group	Amplitude
SG1	15 cms
SG2	12 cms
SG3	9 cms
SG4	6 cms
SG5	3 cms

If it is true that the speed increases as the amplitude increases, then we will find that SG1 has the highest speed, and SG5 has the lowest speed, with the other speeds in between:

$$SG1 > SG2 > SG3 > SG4 > SG5.$$

Ebin: Terrific, Anju. That looks like a great experiment design. How about if you guys come over to my house after school? Let's do the experiment there.

Zak: Wait, wait, suppose our guess is correct. As the amplitude increases, the average speed increases. But amplitude is distance, so when you say that when the distance increases, the speed increases, could it be that the time taken is the for a pendulum? I would like to measure the time too.

That evening, after school, at Ebin's house, they conducted the experiment several times, with different pendulums. And they discovered something that surprised them. Anju was the one who noticed it. that Zak's guess was right. Anju formulated it like this:

Regardless of the amplitude or speed, the time taken for one complete oscillation remains the same for any given pendulum.

Ebin: You mean the *period* of the pendulum is constant: it doesn't change, no matter what the amplitude is or the average speed is?

Anju: Is that what it's called?

Ebin: Yes. That's called the *period*. For example, in the beating of the heart, the time taken from one beat to the next is its period. The time from one sunrise to the next is the period of sunrises.

Zak: Wow. But we have also noticed that across pendulums, the period varies. It's as though each pendulum has its own period. I wonder if it is possible to predict the period of a pendulum on the basis of some other factor.

They left this for the next day.

5.4 Predicting the Period of a Pendulum

- Anju: Could the period of the pendulum have anything to do with the weight of the bob? Sounds commonsensical — the greater its weight, the longer the period.
- Ebin: Aha! how about the opposite? The greater the weight of the bob, the shorter the period.
- Zak: Time to make a list of the variables that might influence the period, and test each of them. The weight of the bob is certainly a possible candidate. What about the length of the pendulum?
- Ebin: That seems a very likely candidate, we *must* test it. How about the density of the bob?
- Zak: The weight would depend on density, wouldn't it? Do we need both?
- Zak: Actually, they can vary independently. So we could keep the density constant and vary the weight, or keep the weight constant and vary the density.
- Anju: Oh, okay. How about the colour of the bob? Material of the bob? How about the material of the supporting rod or string? (laughs)
- Ebin: We can't rule out every conceivable variable completely, but, Anju, these don't sound likely. So why don't we stick to just these two: length of the pendulum, and weight of the bob.
- Zak: So how do we design this experiment? Take a random sample of pendulums, measure their period, length, and weight of the bob, and check if there are correlations?
- Anju: There could be a better way. We can take a separate sample for testing each of these correlations. In one, we vary length, and keep the weight of the bob constant. In the other, we vary the weight, and keep length constant.
- Zak: Nice. That takes care of something very important when doing experiments — to vary only one variable, keeping everything else constant.
- Ebin: Tomorrow is a holiday. Why don't you guys come over to my place for breakfast? And we can do the experiments in my room.
- Anju: If your mother is making that spicy tomato chutney, I am game!
- Ebin: My Mom? *MY* tomato chutney is better than hers. Spicier.
- Zak: (laughing) A bit conceited, aren't we?
- Ebin: No, not conceited. Just confident and realistic.
- Anju: Okay, let's test that claim. How about this? Tomorrow, both you and your Mom make your chutneys. Zak and I will taste both and find

out for ourselves which one is better. And just so that you don't bias our judgment, you shouldn't tell us who made which one.

Zak: Isn't that what they call a double blind experiment used in clinical trials?

Lisa (who was listening from the kitchen): No, not double blind, just single blind. Those who taste the chutneys don't know who made which chutney. But the tasters and the judges are the same. So there's no need to withhold information for two sets of people. So it's single blind.

Anju: I don't understand. What's a double blind experiment?

Zak: We'll look it up on the Internet later.

Ebin: Give me a minute.

He looked up 'double blind experiments' and found links to three YouTube videos:

Placebo Effect, Control Groups, and the Double Blind Experiment
<https://www.youtube.com/watch?v=GMqrOdCx4Yg>

Single Blind Studies and Double Blind Studies
https://www.youtube.com/watch?v=KW_saBXQ0ew

Why does the placebo effect work?
<https://www.youtube.com/watch?v=gm02Oid8sbs>

Anju: Thanks, Ebin! I'll watch them at home later.

When they actually did the two pendulum experiments the next morning, they discovered that there was no correlation between the period of the pendulum and the weight of the bob. But there was indeed a correlation between the period and its length:

Law 17: The greater the length of the pendulum, the greater its period.

Anju: When we say 'length of the pendulum', do we mean the length of the string, or the length of the pendulum as a whole, including the bob?

Ebin: Good question. We'll have to do another experiment to find out.

Zak: That shouldn't be difficult. Take two pendulums in which the length of the support is constant, but the bob has different lengths. One can be a key that's about three inches long, and the other a six-inch metal rod. If the relevant variable is the length of the support, their periods would be the same, since their length is the same. But if the relevant variable is the overall length, then the period of the second one would be greater.

Ebin: We should do the converse as well, just to be sure. For this, we keep the overall length constant, but vary the length of the support. We use the same two bobs — the three-inch key and the six-inch rod. To get the same overall length, we adjust the length of the support.

Zak: Okay, let us do that tomorrow. I need to get to my tennis practice on time.

Anju: Just a minute. I want to write down all the generalisations we have arrived at about the behaviour of the pendulum, and also the new law. (Writes.)

a. For any given pendulum, the period is constant, regardless of differences in the amplitude.

b. For any given pendulum, the greater the amplitude of a swing, the greater the speed.

c. In the population of pendulums, the greater the length of the pendulum, the longer the period. (Law 17)

They stared at the three statements for a minute.

Zak: The statement in (b) follows from (a). Do we have to list it separately?

Anju: How does it follow?

Zak: Imagine a pendulum whose period is two seconds. That will be so whether the amplitude is 10cms or 15cms, right?

Anju: Yes.

Zak: What do we mean by the period of a pendulum?

Anju: The time taken for it to complete a cycle, to go from point A to point B and return to point A.

Zak: Okay. If the amplitude is 10cms, the distance covered by one cycle is 20cms. And the period is 2 second. So the speed is $20/2$, that's 10cms per second. Now, if you increase the amplitude to 15cms, the distance covered is 30cms. And the period is still 2 seconds. What's the speed?

Anju: Ah, okay. The speed is 15cms per second. So it's more. I get it now.

Ebin: Cool result!

Zak: Indeed! So when we put these generalisations together, we predict a new generalisation! That is exciting.

Ebin: Is it possible to derive statement (a) from statement (c), I wonder.

Zak: I don't think so. What if both the amplitude and the length have an effect on the period? So the period may vary depending on the amplitude.

Anju: No, Zak! We've shown that the amplitude has no effect on the period.

Zak: That's true. But (c) is not formulated in such a way that we can derive (a) from (c).

Ebin: Oh, okay, I see it now.

5.5 Collision and Gravity

Zak: You know, we've been talking about two things that cause motion. One is collision, and the other is gravity. In a collision, where one object causes the other to change its speed, the force that causes the motion is present *only during the event of collision*. Gravitational force is different; it acts *continuously*. Its action on a stone, whether it is dropped or thrown up, is continuous. And it acts continuously on the bob of the pendulum. So the swinging bob and the stone in the air both keep changing their speed.

Ebin: Yes, that makes sense. Both the bob and the stone go up because of the force of the push. As they go up, their speed decreases, because gravity pulls in the opposite direction.

Anju: Yes, in a theory that assumes that force causes change of speed, that would indeed be the explanation. But in a theory that assumes that force causes change of location, the speed decreases because the force of the push gets used up.

Zak: The speed reduces and at some point becomes zero. So the motion of the bob/stone has a peak, where it comes to a stop. It then starts coming down, because of the continuous gravitational force. As it comes down, the speed increases.

Ebin: For all this to follow from the laws, we would have to state the nature of the different pushes and pulls.

Anju: Let's see. The stone: it stops when it hits the ground. But there is nothing to stop the motion of the pendulum when it reaches the midpoint, so it continues in the same direction. It goes up, reaches the peak, and comes down. It keeps oscillating back and forth up and down. That is so neat!

Ebin: Do the laws also take care of the behavior of the other things we have been talking about? Like the carrom board, and the wheels?

Zak: I think so.

The next day, back in school, when they started talking about motion, Anju brought up something interesting.

Anju: You know, I was reading the theory of motion that the Greek philosopher Aristotle proposed. He lived nearly a century before Archimedes. What I found interesting about his theory is the idea of natural motion.

Ebin: What is natural motion?

Anju: For him, the natural motion of heavenly bodies is in a circle. That is how he explained planetary motion. But the natural motion of a stone on earth is in a straight line. That explains why when we drop a stone from a height, or a billiard ball hits another billiard ball, it travels in a straight line.

Ebin: Fascinating!

Anju: Oh, there's more. Aristotle asked himself, "Why is it that a stone, or a body of water dropped from a height come down, but smoke and steam go up? If a pot of milk topples over, the milk comes down. But if we boil the milk in a pot, the steam goes up. When we light a candle, the melting wax flows down, but the flame and the smoke go upward. Why is that so?"

Zak: I had never thought of that.

Anju: His explanation was in terms of the natural place for things. In Aristotle's theory, the natural place of stone is earth, so a stone thrown up comes down to earth. The natural place of water, a liquid, is down, but above the earth; so milk when it drops comes down, because it is mostly water. A stone dropped in water sinks to the bottom, but an air bubble in water goes up, because the natural place of air, and also of steam and smoke, is above water.

Ebin: So Aristotle's explanation for a stone thrown up in the air coming down is quite different from the way we explain it.

Anju: Yes. In his theory, the stone goes up because the force from the hand gets transferred to the stone, which makes it go up, and then it stops when the force is spent. And it comes down because its natural place is the earth.

Ebin: That's quite an amazing theory. But how do we choose between the two theories? I have no idea.

Zak: I don't have any idea either.

Anju: Me neither. Let's leave it for now.

STOP AND THINK 5.2

Suppose we throw a stone up at angle of 45 degrees.

Would the stone go up along a straight path or a curved path?

When it comes down, would it be along a straight path or a curved path?

If it is a straight path, would it be vertical, or at an angle?

When it comes down, would its speed be constant, increasing, or decreasing?

Would Aristotle's theory allow you to derive your answers to these questions?

STOP AND THINK 5.3

Consider an archer shooting to hit a target five feet above the ground. The arrow leaves the bow at a height of five feet above the ground.

Should the archer aim the arrow:

 directly at the target, expecting the arrow to travel horizontally?

OR

 aim slightly higher, and expect the arrow to go up and come down?

CHAPTER 6

HOW SCIENCE PROGRESSES

Ebin, tired after a long, busy day, went to bed early. He was fast asleep when a loud knocking on his door woke him up. The knocking wouldn't stop, so he sat up, yawning, dragged himself out of bed and opened the door. He was shocked to find four elderly gentlemen at the door, smiling at him.

"May we come in, Ebin?" one of them asked politely.

"Oh, please do!" a surprised Ebin said, and arranged places for them to sit, concealing his immense curiosity about who they were.

The visitors (V) walked in and introduced themselves before sitting down.

V1: My name is Albert Einstein. (AE)

V2: I'm Isaac Newton. (IN)

V3: Galileo Galilei. (GG)

V4: And I am James Maxwell. (JM)

Ebin rubbed his eyes, and stared at them in utter disbelief. These were names he had heard numerous times. It took a while for him to compose himself.

Ebin: You mean *THE* Einstein, Newton, Galileo, and Maxwell?

The four visitors, still smiling, nodded in unison.

Ebin: But that doesn't make sense. All of you died a long time ago. You, Galileo, died in 1642, the year (pointing to Newton) he was born.

GG: True, Ebin, our material bodies disintegrated a long time ago. We are not manifested as atoms, molecules and cells. Our mass has been transformed into pure forms of energy. Surely, you have heard about our friend Einstein's equation $e = mc^2$?

Ebin: Does that mean you are ghosts?

JM: Oh no, not ghosts. There are no ghosts. Why don't you assume that you are in bed, sleeping, and you are dreaming that you are talking to us?

Ebin thought for a moment, and then cleared his throat.

Ebin: Okay, dream people! You obviously came into my dream for a reason. And you are most welcome! But why are you here?

AE: Oh, we just had to come and tell you how pleased we all are with the progress you and your friends have been making in your efforts to construct a theory of motion. We've been watching you as you think and read and discuss stuff.

Ebin: What?!!!

IN: Not just us. Our friend Archimedes too.

JM: And Socrates as well.

Just then, there was another knock on the door. When Ebin opened the door, he found another elderly person standing there, looking somewhat annoyed. He barged into the room, scowling and waving a finger at the four visitors who were already there.

5th V: You left without me! You know I am interested in this stuff. Why didn't you wait for me?

JM: You were fast asleep, Archie, so we left you a note on your door. (Turning to Ebin) Ebin, this is Archimedes. Archie, you know Ebin, don't you?

Arch: Of course, I do. Socrates has been talking non-stop about you and your friends! And Aristotle too.

They heard a soft cough (or was it a laugh?) from the far side of the room, and discovered that Socrates had been lounging on Ebin's bed all this time, silent but with a broad grin.

Soc: Aristotle wanted to come with me, but he is down with a fever.

Ebin: How did Socrates get in here! Is this really happening? This is incredible company! Socrates, Archimedes, Galileo, Newton, Maxwell, Einstein? Wow... Eh... Wait a minute! Have all of you come to tell me that the theory we're constructing is correct?

The elders looked at one another and smiled.

IN: No, Ebin. You have certainly made excellent progress. But we don't think all of it is CORRECT. You will need to make a number of corrections to make the theory better.

Ebin: So you know what the correct theory looks like?

IN: No, no, we don't. We just know that *ALL* theories may need corrections. Including ours.

Ebin: You're saying the latest theory of physics could be wrong?

AE: Well...we are not saying that it IS wrong. All we're saying is that we cannot be completely certain that it is correct. There is a world of difference between the two statements, you know.

Ebin looked at them in awe. The greatest physicists who ever lived were not certain that the theory they helped to build was correct? He was silent, brooding on what he had heard. Maxwell broke in.

JM: Look, Alfie, the child looks a bit lost. I bet he's wishing his friends were here. Shall we bring in Zak and Anju? This chap will feel better, and those two would get a glimpse of what scientific inquiry is.

Ebin: Can you do that? Have them join us here? Oh, please do!

AE: No, problem, Ebin! GG, would you like to do the honours?

Galileo looked pleased. He closed his eyes, muttered something in Latin, and clapped his hands. In the blink of an eye, Anju and Zak were sitting on Ebin's bed.

Ebin: Awesome!!!

Anju and Zak were speechless, in a state of shock. Ebin explained what was going on, and introduced all the visitors to them. They looked on in awe.

Arch: Now that you are all here, let me ask. Do you think the theory of motion that I constructed long ago is correct?

After a few seconds of awkward silence, Zak mustered up some courage and spoke, though reluctantly.

Zak: Sorry, no, we don't think it's correct. Actually, our theory might be a bit better. Let me try to say why. Your idea was that force causes change of *location*. But we think that idea is flawed, and that it would be better to assume that force causes change of *speed*.

Arch: I'm so glad to hear you say that. You know that I was the foremost scientist of my time. Across the world, textbooks and teachers mention my name even now, with great admiration. Yet, you are critiquing what I said. That tells me that you feel free to critique the authorities and experts.

Socs: Way to go! This is exactly why I wanted to meet you.

AE: Now, you said three things in your critique. One was that Archie's theory was 'incorrect'. Another was that it was 'flawed'. And then you said that your theory is 'better'. Do the words incorrect, flawed and worse mean the same thing?

Anju: There seems to be a difference. But I can't quite put it into words.

Ebin: How about this? If a theory is incorrect, we throw it away. If it is flawed, we repair it. But if there are two theories that are both correct, and neither has a flaw, we choose the better one.

Arch: Excellent. Now, you said my theory is incorrect. But what about the theories of these gentlemen — Galileo, Newton, or Einstein? Are their theories correct? Galileo, what do you think? Is your theory correct?

GG: I don't think so. My theory has fewer flaws than yours, and is better than yours, but I wouldn't say that it is the 'correct' one.

IN: Do you want to explain why you think that your theory has fewer flaws and is better than Archie's?

GG: Hmm! Let me see.

He sat silently for a while, staring at the ceiling and gathering his thoughts.

AE: Galileo, don't answer that question, let the youngsters figure it out. Perhaps just give them a hint in that direction.

GG: Right! You talked about force as that which causes change of location, and force as that which causes change of speed, didn't you? Which of these concepts lead to better explanations?

Anju: The second one, force as that which causes change of speed.

IN: Alright. Which of them did Archie here use?

Zak: It was change of location, right, Archie?

Arch: That's right!

IN: Okay, good. That gives us a handle on the issue: a theory that yields better explanations is the better one. Let us take this a step further. What is speed?

Ebin: The rate of change of location.

JM: And how do you measure speed?

Anju: By measuring the distance that something has moved, and measuring the time it has taken to cover that distance. We then divide the distance by the time taken.

IN: So if Anju runs on a circular path of one kilometer, and returns to the starting position in four minutes, her speed is 250 meters per minute?

Zak: Yes.

AE: Has she had a change of location?

Zak: (After a short silence) Oh, oh! No, because she has come back to the initial location. So the change of location is zero.

GG: So if you define speed as the rate of change of location in a stretch of time, and there is no change of location, then the speed is zero. Yes?

Zak: Yes. That is right.

Anju: Oh no!! That means we're back where we started.

Zak: What do you mean?

Ebin: I see what Anju means. Speed should be defined in terms of distance covered, not in terms of change of location.

Anju: Thanks, Ebin! We solved Zeno's arrow problem by assuming that speed is change of location in a stretch of time. And we also assumed that time is a line made up of very short stretches. But we now have a new problem: if there is no change of location, speed is also zero. But does that mean there is no motion?

GG: Good question, Anju. Let me give you a hint. The concept of speed is not enough for the study of motion. We need another concept.

Anju: Like what?

GG: Okay. A pendulum moves from point A in one direction, passes through the midpoint in its path, gets to point B, reverses its direction, and passing through the midpoint again, returns to A. In your discussion, you wanted to check the speed at that midpoint. So, take that midpoint as point P. Now, the speed of the bob at P would be zero, right? So if you define speed as rate of change of location, as Zeno said long ago, the speed of any pendulum at any point is zero!

Ebin: Oh, the pendulum problem looks exactly like the arrow problem!

Arch: (laughing) He's got it!!! Actually, you can avoid that problem if you have a concept of force that combines speed with the direction of motion.

Ebin: Oh, yes, that must be velocity! I've read about it. Speed in a given direction.

Zak: Ah, I see now! If Anju keeps running at constant speed on a circular path of one kilometer, and returns to the same point in four minutes, there is no change of location between her starting point and end point. But during every small interval of her running, there is velocity in a particular direction.

Ebin: Fantastic! This is the concept we were after when we tried to define force as that which causes a change in speed. We should be defining force as that which causes change of velocity.

GG: That *WAS* my definition of force. Archie's concept of force was as change of location. So, you now see that my concept of force, which is the same as what you implicitly had, is better than Archie's concept of force?

Ebin: That's beautiful, Galileo. So our understanding of the world, including scientific understanding, gets better when we explore the consequences of the concepts we have in our theory. Definitions of the concepts, actually.

GG: Exactly.

Anju: Wait a minute! Wouldn't the Zeno problem disappear — for the pendulum at least, if we define speed as distance covered in time? When a pendulum moves in one direction, then reverses direction, and returns to its original location, there is no change of location. But there is still distance covered. So speed can be non-zero even when the change of location is zero. So why do we need the concept of velocity? Won't speed be sufficient?

Ebin: Excellent point, Anju. Galileo, could you respond to Anju's objection?

To his surprise, Ebin noticed that Galileo was fading. He turned to look at the others, and they too were fading. Then he remembered that it was all a dream.

But Ebin didn't want the dream to end. "Wait, wait," he said, "Don't go yet. I have more questions." But they faded away completely. And Ebin woke up.

It was still early in the night, so he phoned Anju and Zak.

Ebin: Anju, Ebin here. Put your phone on speaker mode so Zak too can hear.

Anju: Done.

Ebin: You know what, I just had a dream in which I talked to Archimedes, Galileo, ...

Anju: (interrupting excitedly) You too? This is amazing. I also had a dream just now in which I talked to Archimedes, Galileo, Newton, Maxwell and Einstein. And when I woke up, Zak told me that he had the same dream. Actually, I was about to call you.

Zak: Ebin, did your dream end with you saying, "Wait wait, don't go yet, I have more questions"?

Ebin: I can't believe this. They appeared to us in our dreams at the same time, and we had exactly the same discussion?

Anju: I want to talk about this concept of velocity.

Ebin: Let's do that tomorrow. I am still in shock. And I'm tired and need to go back to sleep.

Zak: Tomorrow, then. Good night, Ebin. Rest well.

Ebin: Good night, Zak. Good night, Anju.

CONCLUDING REMARKS

We hope you enjoyed reading this book, that you have learnt something from it, and that the *learning* experience has been joyful.

For your learning to be meaningful, there are two things that would be useful for you to do. The first is this. Every few days, perhaps even at the end of each day, ask yourself: What did I learn today, this week, this month, this year that I didn't know before? Identifying what you learnt is not easy at first, but with practice, you will learn to do it, and do it well. And you will grow as a *conscious learner*.

Once you have identified what you learnt, ask, "Where did I learn it? What did I learn it from?" and "How did I come to learn it?" If it was from a book (or some reading), reread it. Now go through that process of identifying what you have learnt in the second round of reading. And then read it again, to go deeper and broader. Each time, you will discover deeper layers of learning — and maybe some details that you may have missed in your first round. You will also have a better understanding of what the book is offering you.

As you think about the phenomena of motion discussed in this book, and the theoretical explanations for them, keep in mind that the motion of inanimate things is only one kind of motion. We have not explored the motion of bacteria, of cells in the living body, or of plants, worms, snakes, fish, butterflies, dragonflies, birds and horses. And even within the motion of inanimate things, we have not explored the motion of planets, stars, and galaxies on the one hand, and of molecules, atoms, and fundamental particles on the other. Nor have we explored the motion of rivers and of clouds; or of a boat, of a piece of wood rising up to the surface from the bottom of a lake, of air bubbles rising from the bottom of a pan of boiling water; of a feather in the breeze, or a dry leaf falling from a tree; or of paper planes that children make. You may have not explored changes that don't come under motion, or phenomena that come under system, structure, and function, not under change. The world we live in is full of fascinating phenomena that trigger puzzlement, if you are curious about them. These phenomena can provide anyone a lifetime of joyful learning, and for a research student, a lifetime of exploration.

The second requirement for meaningful learning is a sense of what a book hopes you would learn from it. To some extent, you may have figured out the learning that this book aims at, from the remarks made by the characters in the dialogues: Ebin, Anju and Zak, and Ebin's mother, Lisa;

and also the night time visitors in Chapter 6: Socrates, Archimedes, Galileo, Newton, Maxwell and Einstein. In addition, there are the remarks made as side comments by the authors.

Nevertheless, let us articulate our aims explicitly, so that you can check how well the chapters in the book help you achieve those aims.

What this book aims at has two components:

- a conceptual understanding
- and based on that understanding,
- a set of abilities, capacities, attitudes, and habits of mind needed for exploring patterns in the world around us, and for building theories..

The understanding component itself has two parts. One is understanding of the nature of scientific knowledge, and of human knowledge in general. The other is an understanding of the concepts needed for constructing theories of the motion of inanimate bodies, called mechanics in the physical sciences. We hope that the conceptual understanding provided in this book would form the foundation for any advanced study of science you might pursue, and that your study would be deeper and more valuable.

The second component includes the ability to:

- A. identify and formulate questions or problems to investigate;
- B. look for answers/solutions, using experience (including sensory experience), and a combination of observation and reasoning;
- C. arrive at and critically evaluate answers and conclusions;
- D. go beyond (C) where needed, and consult documented sources of knowledge available on the Internet (Wikipedia, YouTube, and so on), in libraries, and from learners with more experience;
- E. engage critically with the answers and conclusions, and defend one's conclusions to the satisfaction of a jury of fellow inquirers; and
- F. engage in debates where needed, for collective learning.

Underlying these abilities is a deep intellectual curiosity, a greater awareness of worthwhile questions and problems to pursue, and the joy of discovering both the questions and answers to them.

In the outline given above, we have left out something fundamental that we hope you have learnt. Go through Chapter 6 again, think about what we have left out, and write it down. Here is a clue: even if you forget about everything else, we hope that what you receive from Chapter 6 will become part of you, and remain with you throughout your life. We hope that this book will be the start of a never-ending journey for you.

If you have comments, questions, criticism, or feedback on any part of this book, do contact us. We would love to hear from you.

A THEORY OF MOTION:
APPENDICES
FOR THE ADVANCED READER

CONTENTS

- 1 Doubting and Questioning
- 2 Physics of Motion before Galileo
- 3 Archimedes' Levers: Math or Science?
- 4 Galileo and the Pendulum
- 5 From Verbally Stated Laws to Equations
- 6 Is Friction a Force?

1 DOUBTING AND QUESTIONING

The dialogues and tasks in Chapter 1 are meant for readers who have not yet been exposed to Newton's Laws of motion. If you are familiar with Newton's Laws, you may be concerned about the principles that the young inquirer-researchers, Anju, Zak, and Ebin, have constructed.

For instance, the investigation of motion in this Chapter is restricted to the concepts of *weight* (as distinct from *mass*), and to *speed* (as distinct from *velocity*). This move is necessary if we subscribe to the pedagogical principles of ***experiential learning***:

- a. What children learn must be rooted in their experience.
- b. When they reject their experiential knowledge, and accept the statements that are closer to current received knowledge, they should understand the reasons — the evidence and arguments — for doing so.

For example, our experience tells us that the earth is flat, and completely stationary. When rejecting that position, and accepting the statements that the earth is round, and that it rotates, and revolves around the sun, learners should understand the reasons for doing so. Requiring learners to accept textbook statements without giving reasons is a form of ***indoctrination***, a form of education that the authors of this book do not subscribe to.

You might wonder why the children come up with principles and concepts that modern science would judge to be 'incorrect'. For instance, the concept of force in Newton is that which causes *acceleration: a change in velocity*; while the children's concept of force is that which causes *motion: a change of location*. The advanced reader might wonder: "Shouldn't we correct their 'misconception'?"

The aim of this book is not to transmit to students a body of statements that the current research community judges to be 'true'. , Rather, it seeks to help learners become inquirers and researchers (as stated in 'About the Book': p. 5) — to develop the ability to ***construct knowledge*** and to ***evaluate knowledge claims***. So, even when they construct principles/laws that are considered 'incorrect' by standard textbooks, the way they arrive at these laws illustrates modes of constructing knowledge, and provides them with practice. And when they reject their own formulations on the basis of evidence, they learn modes of critical thinking and self-correction.

In other words, the concepts they come up with are not misconceptions; they are grounded in their own experience, and they support them with rational arguments.

The theory of motion that Galileo and Newton developed falls within ‘classical mechanics’. What the young inquirers in our dialogues develop is a ‘pre-classical mechanics’, in harmony with the ideas developed by researchers before Galileo, say, in Ancient Greece. ‘Pre-classical mechanics’ assumes that *force* is that which causes motion, and *inertia* is that which resists motion. It also takes for granted what is called the *mechanical* concept of causation, which assumes that in order for X to cause an effect in Y, X and Y must be in physical contact. This principle prohibits the formulation of laws that involve what is called “action at a distance”.

The Principles of Motion in this Chapter are *tentative*. In later chapters, the children, who in effect form a community of inquirers and novice researchers, might find reason to revise these principles, or to reject some of them as false.

These children already exhibit a sense of uncertainty and fallibility of the knowledge they have constructed. Towards the end of the book, they discover that leaders of the world of science have had the same awareness of the *uncertainty and fallibility of human knowledge* itself. And that in turn prompts a predisposition to *doubt and question* what is presented to them as knowledge. These attitudes and predispositions are fundamental to scientific inquiry.

In addition to the attitudes and habits of mind, we distinguish the following pedagogies, relevant for both a classroom and a textbook:

A. Exposition-Based Pedagogy

The teacher/author presents a spoken or written exposition from which students learn. The primary goal for the student is understanding.

B. Interaction-Based Pedagogy

- a. Teacher-Student Interaction: This is best done in a one-to-one interaction, where the student learns by asking questions, answering the teacher’s/author’s questions, and so on. This is the mode used in the so-called Socratic dialogues (written by Plato), as well as other dialogues (e.g., Galileo’s dialogues).
- b. Peer Interaction: Students interact in small groups of four or five. When such groups work together in the classroom, we call them buzz groups. When the group work happens outside the classroom, we call them affinity groups.

Interaction-based pedagogy can be practiced in the online mode as well, for example, through zoom sessions, and does not exclude exposition.

The pedagogy used in this book is that of peer interaction in a virtual mode. The students who engage in the interaction are not the readers (or viewers, in the case of videos), but the characters in a fictional set up. But the readers/viewers interact vicariously in their mind, and get a sense of what it is like to collectively engage in interaction to acquire construct and evaluate knowledge.

2 PHYSICS OF MOTION BEFORE GALILEO

If you have taken an advanced course in physics, you would recall that in addition to the laws of linear motion, Newton also formulated laws of rotation. How do we integrate these two sets of laws into a single theory?

As stated in Appendix 1 ('Doubting and Questioning'), this Chapter and the one before it are located in pre-Galilean classical mechanics. It is only by understanding what phenomena this theory seeks to explain, and how it explains them, that we really understand Galilean-Newtonian classical mechanics, and only then should we proceed to areas such as relativity and subatomic physics.

You might want to compare the theory emerging in these chapters to the theory that you have learned in physics before.

3 ARCHIMEDES' LEVERS: MATH OR SCIENCE?

After their discussion of seesaws and levers, when Ebin got home, he did a YouTube search for levers, and discovered a video lecture by mathematics professor Norman Wildberger. It was called: "Archimedes and the Law of the Lever Module 1: Levers and Centre of Mass." (<https://www.youtube.com/watch?v=SncS82GBOQc>) It was just 13 minutes long. He was delighted to find that the law of levers in equilibrium that he, Zak and Anju had come up with was what Archimedes had formulated more than two thousand years ago. Also, there were other laws for other types of levers. He immediately shared the URL with Anju and Zak, asking them to watch the video.

When they met the next day, they had all watched the video more than once.

Ebin: I have a question. Professor Wildberger is not a physicist. He works in the Department of Mathematics and Statistics. And Archimedes was a mathematician, scientist, engineer and inventor. What I'm wondering is, how did he arrive at the laws? He was thinking like a mathematician, wasn't he? Not like a scientist or an engineer.

Zak: I had a similar feeling. Archimedes did some thought experiments, arrived at some premises, and deduced the laws from those premises. That is what mathematicians do. He came up with the laws, but he didn't bother to do any experiments to check if the laws were correct. To that extent, it's not science.

Anju: Isn't that a bit extreme, Zak? Einstein did exactly what Archimedes did. He did thought experiments, thought about their results carefully, and arrived at conclusions based on those results. Those conclusions are his theory of relativity. He didn't do any experiments either. But he is still a scientist, isn't he?

Lisa had come in, and was listening to the conversation.

Lisa: Yes, of course, Anju. There were others who did experiments to test Einstein's theory. Einstein was a theoretical scientist, not an experimental scientist. But he definitely was a scientist.

Anju: We're talking about theoretical science vs. experimental science. You seem to be saying that there was philosophy, mathematics, and theoretical science during the Ancient Greek times, but experimental science only came later?

Ebin: As far as I know, experimental science began with Galileo.

The children were mulling over that when the bell rang.

4 GALILEO AND THE PENDULUM

The correlation between the length and the period of a pendulum (**Law 17: *The greater the length of the pendulum, the greater its period***) is essentially what Galileo came up with at the beginning of the 17th century. Readers who are familiar with Galileo's law of the pendulum might find it useful to watch the video, "The Pendulum and Galileo," at: <https://www.youtube.com/watch?v=MpzaCCbX-z4>

The video places Galileo's experimental methodology and findings in a broader historical context, providing a contrast to the intellectual climate of the Middle Ages, especially that coming from Aristotle's teachings.

It is important to pay attention to this transition in the evolution of scientific thinking and scientific knowledge. It also provides an introduction to the statement of the law of the pendulum first in purely qualitative terms, as Anju, Zak, and Ebin did. It then slowly builds up to a translation of that statement into an algebraic notation, as the first step towards a quantitative formulation of the law.

While viewing this video, do pay attention to how “is directly proportional to” is expressed as the symbol \propto in $T \propto l$, and how the quantitative formulation in terms of the equality symbol ‘=’ allows us to make precise quantitative predictions.

As this video makes clear, Galileo’s contribution to physics was far more than the law of the pendulum, drawing attention to:

- (i) the careful designing of experiments to separate the different causal variables;
- (ii) the invention of a system of measurement; and
- (iii) the painstaking implementation of the system.

Pre-Galilean physics was largely based on non-experimental observations, and explanations that used mathematics. Counting the number of pendulum oscillations for 24 hours a day, Galileo laid the foundations for experimental inquiry as an integral part of science.

Another video on the subject is Professor Julius Summer Miller’s “Lesson 7–The Simple Pendulum, Oscillating Things–Demonstrations in Physics,” at: https://www.youtube.com/watch?v=h_eR7FJ_vkM&t=6s

It is part of a set of physics videos that can be found at:

<https://www.youtube.com/user/dramaticphysics/videos>

In the video by Professor Miller, do pay attention to the reference to ‘ideal rigid support’ and the ‘ideal pendulum’, where ‘ideal’ means *a state in which all the non-essentials have been left out*. (This point came up in a discussion between Ebin and Lisa in Chapter 2.) Such idealisation is an integral part of mathematics and theoretical science. Professor Miller also points attention to some important aspects of experiment design.

One final remark: You may have come across the terms *velocity*, *momentum*, *acceleration*, and *mass* in the discussion of motion in your textbooks. The theory of motion developed by Anju, Zak and Ebin does not use these terms. This is because the theory they are developing is within a conceptual framework of physics before Galileo. They do move from the concept of force as causing a *change of location*, which is an Aristotelian concept, to the concept of force as causing a *change of speed*, which is halfway to Galileo. The Galilean concept of force as causing a *change of velocity* would be the next step, but we reserve that for a slightly more advanced discussion.

5 FROM VERBALLY STATED LAWS TO EQUATIONS

[Continued from the end of Chapter 3]

When Anju, Zak, and Ebin met the next day, Anju said:

Anju: Listen, can we go through the stuff we talked about before we went home yesterday? Ebin, you started it, something about stating the laws not as sentences but as something that looked like equations. Our laws in words were:

Law 12: *The greater the distance between the fulcrum and the object, the greater the force required for lifting the object.*

Law 13: *The greater the distance between the fulcrum and the location of the force applied, the lesser the force required for lifting the object.*

Law 14: *The greater the weight of the object, the greater the force required for lifting it.*

And you used the symbols F, D, and W to express these laws. And also the proportionality symbol \propto .

Ebin: Oh, but they are not equations. They don't have the 'equals' symbol.

Zak: Okay. According to the Law 12, if we take the distance between the fulcrum and the object to be D, and the force required for lifting the object to be F, then the law says that the value of D is *directly proportional* to the value of F, right? And we stated this as:

$$F \propto D$$

Ebin: Right. And Law 13 says that F is *indirectly proportional* to D², which is the distance from the fulcrum to the location of the force. And we stated it as:

$$F \propto 1/D^2$$

Anju: Oh, just so that there is no confusion, we should refer to the distance in Law 12 as D₁, and write it as:

$$F \propto D_1$$

Zak: What about Law 14? We didn't finish talking about it yesterday.

Anju: The law talks about the weight of the object being directly proportional to the force. If the weight of the object is W, then the statement would be:

$$F \propto W$$

Ebin: We haven't said anything about what happens when there are two objects, one on each side, and the seesaw is horizontal. Remember, that is how the whole thing started. Zak was sitting on a seesaw, and when Anju sat on the other side, her side went down even though Zak weighs more than Anju. The laws that we have formulated so far don't tell us when the two weights on a seesaw would be balanced.

Zak: Oh, yes. That's true. We still don't have a theory that explains all that we want to explain. Let us think about it some more and discuss it tomorrow.

The next day, when the children met in school, they found that they couldn't make any progress. So after school, Ebin suggested that they go with him to his home, to ask his mother, Lisa, for help.

Lisa listened to them attentively, and smiled.

Lisa: Listen! If I tell you what I think the theory is, you will miss a chance to build it yourself, won't you? And if I don't help at all, you may get stuck, and not be able to progress. So what I should do is help just a wee bit, with hints and clues. And you do the rest yourself. Agreed?

Ebin: Mom! Please! First clue?

Lisa: Okay. Let us try this. Suppose two of you are sitting on either side of the fulcrum, and the seesaw is balanced. It stays horizontal. If your weights, say W_i and W_j , are the same, your distance from the fulcrum, D_i and D_j would also have to be the same, right?

Anju: Right!

Lisa: But if W_i and W_j are not the same, what would D_i and D_j be?

Zak: Let me try. Suppose we decrease the distance on one side. Then the seesaw will be unbalanced, right? The side with decreased distance will go up. To make it balanced, we will have to increase the weight.

Lisa: Keep going, Zak, you're going in the right direction.

Zak: Ah! I get it. The object on each side does two things. Its downward force is proportional to its weight, but it also has an upward force on the object on the other side. And that is proportional the distance on each side!

Ebin: Oh, yes. So we need to say something about the relation between the weight and distance on one side, and the weight and distance on the other side.

Anju: Right. Balanced means that the weight and distance on one side will be the same as weight and distance on the other side.

Zak: Weight and distance? Does that mean we add the two, or multiply them? If it is addition, the law would look like this:

$$W_i + D_i = W_j + D_j$$

But if it is multiplication, the law would be:

$$W_i \times D_i = W_j \times D_j$$

Ebin: Why don't we keep both for now, and do some experiments? Only that will tell us which law works.

Anju: Hey, those laws have the equals symbol, so they are equations, just like the laws in our textbook!

The children felt triumphant, and looked at Lisa, who was beaming.

6 IS FRICTION A FORCE?

Ebin: When we strike a carrom piece, it slows down and then it stops. What is the force that changes its speed?

Zak: Aha! I got it. It has to be friction, a force that happens when two things rub against each other.

Ebin: Friction? Is friction a force?

Zak: Well, friction must be a force. We have assumed that only a force can cause a change of speed, and that anything that causes change of speed is a force. And since friction causes change of speed, it must be force.

Anju: Nice!

Zak: Okay, try this thought experiment. Imagine placing a carrom striker on a rough concrete floor, and striking it with a given force. It will go some distance, and then stop, right? We assume that this is because of friction, which is a force that opposes motion.

Ebin: Okay.

Anju: So?

Zak: Now imagine placing that striker on a huge carrom board as big as a stadium, and striking it with the same force. Will it travel the same distance on the carrom board as the one on the concrete floor? Or will it travel a greater distance? Or a lesser distance?

Ebin: It will travel much farther on the carrom board than on the floor.

Zak: And why is that?

Ebin: Aha! I see now. Because the surface of the carrom board is smooth, and the concrete floor is rough.

Zak: Right! What you're saying is, there is less friction on the carrom board surface than on the concrete floor. Now imagine that you are placing the striker on an ice-skating rink.

Ebin: Oh, there it will travel much farther, because the ice-skating rink has very little friction.

Zak: Exactly! Now imagine an ice-skating rink that is infinite in all directions. And imagine that it has no friction. Absolutely no friction whatsoever. How far will the striker travel?

Ebin: Oh! Oh, my goodness! It will just keep going on and on at constant speed in a straight line. It will go on forever.

Zak: Right! That is exactly what our revised Law 1 predicts. It says that inanimate entities that are not manmade change their speed only when an external force causes them to do so. So when an inanimate entity is in motion, it will go on forever at a constant speed, unless some force causes it to change its speed.

Ebin: Amazing. So if there is exactly one tennis ball in the world, no planets, no stars, no asteroids, not even air molecules anywhere, and the tennis ball is moving, it will keep moving at constant speed FOREVER!!!

All three of them fell silent, and sat contemplating the consequences of the beautiful theory they had created.

