CONSTRUCTING, MEASURING, AND CALCULATING: Fun Questions to Engage with

K P Mohanan and Tara Mohanan

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The primary goal of this document is to give the readers an experiential understanding of the distinction between *mathematical inquiry* and *scientific inquiry*. It also has a secondary goal: to provide an experiential understanding of the distinction between the *geometry of flat surfaces* and the *geometry of spherical surfaces*.

This is not meant for total beginners. Even though many of the activities in this document do not require any prior background other than what readers who have completed eight years of school education can be expected to have, a few of them require a rudimentary understanding of the tools and modes of inquiry that the learning resources at ThinQ (https://www.thinq.education/hoc) offer.

TERMINOLOGY:

Straight Edge: If you fold a piece of paper from one side to another, you get a straight edge.

Ruler: A ruler is a measuring instrument with a straight edge of fixed length divided into equal lengths. For example, a foot ruler is one foot long, and it is divided into twelve equal parts called inches, each of which is divided into ten equal parts. A meter ruler is one meter long divided into hundred equal parts called centimeters, and each centimeter is divided into ten equal parts called millimeters.]

Radius: The straight line AB (in task A) is called the ***radius*** of the circle.

Diameter: Line AC (in task B) is called the ***diameter*** of the circle.

ACTIVITY 1

Task A: Mark two points A and B on a piece of paper. With a ruler and a pencil and draw a straight line from point A to point B. [Place the straight edge of the ruler against A and B, place the tip of a pencil on A, and move it along the straight edge until you get to point B.]

Task B: Find the length of line AB.

Task C: Do the same thing again, but this time with a straight edge which is not a ruler. Is it possible to find the length of line AB?

Task D: If your answer to task C is 'no', what do you think is the reason? [Why is it that you can do task B but not task C?]

Activity 2

Task A: Mark two points A and B on a piece of paper. Draw straight line from A to B, with a straight edge and a pencil. Now draw a circle with a pencil and a compass, with A as the center and B on the circle.

Task B: Draw a straight line from B through A, and continue until the line meets the circle. Call it point C.

Task C: Find the length AB and AC in the circle in Task A.

Task D: Take a point D distinct from A and C on the circle in task A, and draw a straight line from D to the center A, such that DA is a radius. Extend DA till it meets the circle at a point. Call it E. DE is now a diameter of the circle. Find the length of DA and DE.

Do BA and CA have the same length?

Do BA and DA have the same length?

Do BC and DE have the same length?

If someone did not believe your answers to the above questions, how would you convince them that your answers are correct? (i.e., how would you prove that your answers are correct?)

Activity 3

Mark an arbitrary point G on the circle, distinct from A, C, E, and F. Then draw the longest possible straight line GH such that H is also on the circle.

Is GH a diameter of the circle?

How would you prove that GH is a diameter, no matter where you choose G?

Do the longest straight lines between any two points in a circle have the same length or different lengths? How would you prove that your answer is correct?

Activity 4

Task A: The flat surface of a piece of paper has finite size, and a line drawn on that surface, from one edge to the opposite edge, the line is of finite length. Now imagine a flat surface of infinite size such that a straight line drawn on that surface can be infinitely long (drawn from point A to point B, and extended as a straight line that never ends).

Task B: Now imagine points A and B on that surface such that they are more than 10,000,000 kilometers apart. And imagine a straight edge that is infinitely long. In your mind (not with your physical hands), draw a straight line from A to B with that straight edge.

Task C: Taking the line in task B, draw a circle with A as the center and AB as the radius.

Task D: Is it possible for you to find the length of the radius or the diameter of the circle in task C? State the reason(s) for your answer.

The tasks in Activities 1-3 combine ***hands-on*** activities (i.e., activities that you engage with using your hands in the physical world) and ***mind-on*** activities (those that you engage with in your mind, using your imagination). Activity 4 focuses on purely mind-on tasks, with no hands-on component.

If the radius of the circle that you have constructed in your mind through your imagination (with imaginary pencils, imaginary straight edges, and imaginary compasses) is more than a certain length (as in task C), you cannot find out the length of the radius, diameter, or circumference of the circle by ***measuring*** it.

However, if the value of one of these, say the radius, is given to you, you can ***infer*** the value of the other (the diameter or the circumference) by using a ***formula***, and ***calculating*** the length**.**

ACTIVITY 5

Task A: Repeat Activity 3, this time, using only your mind. The difference is, this time, think of the circle as having a radius of more than 10,000,000 kilometers.

Task B: Has engaging with the tasks in Activity 4 given you a better understanding of the distinction between scientific inquiry and mathematical inquiry? Describe what you have understood.

ACTIVITY 6

Task A: In Activity 1, we suggested a way of constructing a straight line from point A to point B using a pencil and a straight edge. Instead of a straight edge, is it possible to construct a straight line using a thread or a rope? Do this as a mind-on activity. If your answer is yes, describe how you would do it.

Task B: Think of how you would construct a circle, using a thread, a thumbtack (drawing pin), and a pencil; and how you would construct an ellipse, using a thread with its ends tied into a loop, two thumbtacks, and a pencil. If you are stuck, take a look at Appendix 1 for a few hints, but not until you have given a serious chance to your creative imagination to come up with ideas.

ACTIVITY 7

Imagine a watermelon with a perfectly spherical surface. How would you construct a straight line on the surface of that watermelon?

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Activity 8

Task A: "If you construct a straight line on a flat surface, the two ends of the straight line would never meet, no matter how long the straight line is. Check if the above statement is true in your mind.

Task B: "If you construct a straight line on a spherical surface, the two ends of the straight line would always meet if it is sufficiently long." Check if the above statement is true in your mind.

> If you find it difficult to imagine, take a physical object that is spherical (such as a watermelon or a lime) and proceed to the imaginary spherical surface after your experience with the physical sphere through hands-on learning.
>
> If you still find it hard to imagine, try this. Take a lemon (or some other spherical object) and cut it into two equal halves, with a flat knife passing through the center. Put the two halves together. Is the line on the surface, between the two halves, a straight line? Check using a stretched thread.

Activity 9

If you construct two straight lines on a flat surface, the two straight lines can intersect once, but never twice.

Check if the above statement is true in your mind.

But if you construct two straight lines on a spherical surface, the two straight lines would always intersect at two distinct points if they are sufficiently long. (But never more than twice, though.

Check if the above statement is true in your mind. If you find it difficult to imagine, take a physical object that is spherical (such as a water melon or a lime) and proceed to the imaginary spherical surface after your experience with the physical sphere through hands-on learning. You may use the strategy of using a thread or the strategy of cutting object through the center.

Activity 10

On a flat surface, construct a triangle:	~	with a right angle;
and if you can, a triangle:	~	with two right angles?
On a spherical surface, construct a triangle:	~	with a right angle;
and if you can, a triangle:	~	with two right angles;
	~	three right angles.

Appendix 1: Geometric Constructions

(with a thread, thumbtacks, and a pencil)

Hints

1: If you let a piece of thread fall on the floor it is not straight, but if you hold the two ends and stretch it, it is straight.

2: Imagine a thumbtack on a piece of paper on a flat drawing board, and a thread attached to it at one end, and a pencil at the other. Now pull the pencil away from the thumbtack such that it is stretched. Place the tip of the pencil on the paper, and drag it while keeping it stretched. What geometric object would you be drawing?

3: Tie the two ends of the thread together such that it forms a closed loop. Place two thumb tacks on a piece of paper such that if you put the loop around the thumb tacks loosely, it is not straight, but if you pull at a point on the loop, it forms a triangle with the thumb tacks as two of its vertices. Now place a pencil on the point where you are pulling, and drag it along the paper while keeping it stretched. What geometric object would you be drawing?

4: (3) above uses a thread triangle to construct another geometric object. Does that suggest a conjecture that expresses a correlation between triangles and the other geometric object? Can you state that conjecture?

Appendix 2: Surfaces in Geometries

Flat vs. Spherical

The geometry that students learn in school is the geometry of flat surfaces, in contrast to the geometry of spherical surfaces.

In flat surface geometry:

No straight line, however extended as a straight line, can meet itself,

No two straight lines, however extended as straight lines, can intersect at two distinct points.

No triangle can have two right angles.

In spherical surface geometry, however,

Every straight line, when extended (as a straight line), meets itself,

Any two straight lines, when extended (as straight lines), intersect at two distinct points, but not more than two points).

A triangle can have two or three right angles (neither less than two nor more than three.)

The specific form of flat surface geometry that students learn in schools is Euclidean Geometry, invented by Euclid in the century BCE. The most well-known spherical geometry is the one invented by Riemann in the 19th century.